Price Signaling with Information Acquisition

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Abstract

We study a market in which the buyer has no information about product quality, while the seller has private probabilistic information about it. Buyers observe price and can procure an inspection, which provides valuable information about the good for sale. With costless inspections, there is no separating equilibrium. We then show that when information acquisition is costly, there is a separating equilibrium that satisfies the intuitive criterion, in which high prices signal high quality. Finally we discuss the implications of time-on-the-market on separating equilibria. Specifically, when there is only one asset on sale over both periods (therefore both price and time-on-the-market may signal quality) there is no separating equilibrium even if single-crossing is satised. The key to this result is that the second-period buyer cannot observe why the asset did not sell in the first period. Notably, the failure to sell can be attributed to overpricing or an unfavorable inspection outcome. Therefore the copycat behavior is more attractive to the poor-quality seller because he benefits more from an increase in buyer beliefs than his high-quality counterpart. Allowing only the first-period buyer to acquire information on quality, we show the existence of a separating equilibrium in which high prices and time-on-the-market signal high quality.

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1 Introduction

Information acquisition is a common activity before making a purchase. Since consumers often cannot ascertain the quality of the product for sale, they are willing to invest time and money on inspecting the good before purchasing it. Inspections are pervasive in the real estate, car and art markets, where prospective buyers spend significant resources trying to avoid costly mistakes. Since inspections are costly and imperfect, agents are strategic in their use. The price of an asset as well as the prior assessment of its quality influence the willingness to pay for information acquisition. High-priced items are more susceptible to be inspected, but also pessimistic beliefs about the item's quality encourage the gathering of information. When the seller is better informed about product quality than the buyers, prices might be used as a signaling device by good-quality sellers. This is so because high prices encourage information acquisition, and inspections are more costly for low-quality sellers (it is more often than their products are found wanting). We characterize the optimal pricing strategy for high-quality assets when information acquisition prior to purchase is possible.

We set up a model with asymmetric information regarding quality and information acquisition, in which the buyer has no information about the asset quality, while the seller has private information about the probability of owning a high-quality asset. Buyers observe prices and can procure an inspection, which provides valuable information about the asset for sale. The seller's pricing strategy provides a signal from which buyers can infer its type, even as it determines the precision of the information regarding product quality they can acquire. Higher prices lead to higher product exposure via inspections, so that prices become an instrument through which the seller may encourage or discourage information acquisition. We look for separating (pure-strategy) equilibria and apply the intuitive criterion refinement (Cho and Kreps 1987) to eliminate implausible off-equilibrium path beliefs.

We first consider, as a benchmark, the case with costless inspections, and show there is no separating equilibrium. Any strategy that is profitable for the high-quality seller can be imitated by the low-quality one, who just sells less often. We then consider the case in which the buyer might increase his signal precision, through costly information acquisition. Here, we show there is a separating equilibrium in which high prices signal high quality. The intuition is that high prices induce more information acquisition, which is incentivized by sellers confident enough that inspections will yield good news and therefore will not decrease demand substantially.

We finally discuss the implications of time-on-the-market on separating equilibria, when the selling season is composed of two periods and there is only one asset on sale over both periods. Then history becomes important since it carries significant information. When facing a decision in the second period, the buyer knows that the first-period one chooses not to buy for one of two reasons. Either the common component signal was bad (which is very relevant for him) or the private component was low (which is irrelevant). Higher prices put a higher weight on the overpricing explanation, decreasing information transmission. We show that the sufficient conditions for the existence of a separating equilibrium do not always hold. The key to this result is that the good seller sells more often in the first period, ending the game, which in turn makes the poor-quality seller more interested in the reputational effects induced by a no-sale in the first period. Since this is achieved through high prices, the possibilities of mimicking the good type are enhanced, making separation often impossible. Finally, we show the existence of a separating equilibrium in which high prices and time-on-the-market signal high quality, when only first-period buyer is allowed to acquire information. In this case, the second-period buyer learns about quality from observing the price history as well as the past purchase decisions.

Literature Review. The model developed here is related to the literature on signaling (high) quality through (high) prices. Bagwell and Riordan (1991) show that high (and declining) prices signal high quality, in a monopoly market for durable goods, in which quality is correlated with costs. Judd and Riordan (1994) reach the same result by examining a two-period signal-extraction model with learning. Even though no correlation between quality and costs is assumed, private information on both sides of the market allows the seller to signal high quality through high prices. Neither costly information acquisition nor time-time-on-the-market play a role in achieving an effective high-price signal. In both models, imperfectly informed consumers may interpret signals to effectively improve information, but inspections or other forms of information acquisition are not available. This paper is also closely related to the literature on signaling with information acquisition. For example, Bester and Ritzberger (2001) consider a static model in which an informed monopolist chooses the optimal pricing strategy for an asset of unknown quality to consumers. Consumers can infer quality from the price or pay for access to an external source of information. For small costs of information acquisition, there is no separating equilibrium in pure strategies, which confirms the Grossman-Stiglitz paradox. Prices cannot be informative, because if they were, no one will pay for information and a low-quality seller would mimic the high-quality one. Furthermore, they show there is a unique partial pooling equilibrium, that resolves the paradox and satisfies the intuitive criterion, which involves mixed strategies and sufficiently small costs of information acquisition. The model presented here achieves a separating equilibrium without the use of mixed strategies or exogenous informational sources. Gertz (2014) examines a monopolistic market with quality uncertainty and information acquisition, a setting very similar to the one presented in Bester and Ritzberger (2001). Nevertheless information acquisition is now endogenously determined, by allowing the buyer to optimally choose the search effort. He characterizes all possible market equilibria and focuses on consumer's behavior and welfare. The main result is that consumer's welfare is maximized at a pooling equilibrium with no search. If the buyer is given the possibility of information acquisition, he can use this search ability as a threat (even if the search proves fruitless), which forces down the equilibrium price. This paper is the closest to the model presented here, even as the focus is on consumer behavior and welfare rather than the strategic actions of firms. Furthermore, time-on-the-market is not considered. Mezzetti and Tsoulouhas (2000) analyze a principal-agent model where the principal is privately informed about his type and the agent could gather information about the principal's type, at a monetary cost, before engaging in a relationship. They find that, if uncertainty is high and the precontractual investigation is not too costly, there exists a separating equilibrium in which a favorable principal is able to separate himself from his unfavorable counterpart. Separation can occur due to the renegotiation option offered by the principal in the worst case scenario that the investigation results in an unfavorable outcome. The idea of signaling with costly information acquisition is present here, but applied to the context of a principal-agent optimal contract problem. Finally we extend the literature on time-on-the-market as sign of quality, initiated by Taylor (1999). Taylor (1999) explores the effect of time-on-the-market on pricing in a two-period model with asymmetric information and a single object for sale (a house). The parametric assumption made about the quality of the item allows him to rule out separating equilibria and focus attention to consumer learning. The main result is obtained without considering information acquisition and involves a pooling equilibrium, in which the low-quality seller mimic his high-quality counterpart. Depending on the information structure, the seller may post a higher or a lower price in the first-period.

2 The Model

We consider a model with asymmetric information regarding quality and information acquisition, in which the buyer has no information about product quality, while the seller has private probabilistic information about it. The quality of the asset may be either high or low, $q \in \{0, 1\}$, and the seller is aware of his type θ, the probability of owning a high quality asset, that can be either good or bad, $\theta \in \{g, b\}$, with $0 < b < g < 1$. The ex post valuation of the buyer is qv, where q represents the common valuation (objective quality) and v is the buyer individual valuation or "taste" for the product, drawn from a distribution $G(v)$ continuously differentiable with $G'(v) = g(v) > 0$, for all $v \in [0, 1]$. The seller's valuation of the asset and his production cost are zero and there is no discounting.

Each period, after observing the price and before making his purchase decision, the buyer procures an inspection on quality. The outcome of the inspection may be either favorable or unfavorable, $s \in \{F, NF\}$ and it is characterized by the following conditional probabilities:

	0	1
F	$1-\sigma$	1
NF	σ	0

A high-quality asset always results in a favorable outcome, whereas a low-quality one generates it with probability $(1-\sigma)$. Therefore a favorable outcome does not guarantee high quality, whereas an unfavorable outcome can be thought of as discovering a flaw in the asset, fully revealing low quality. Note that no buyer will buy the asset if the inspection outcome was unfavorable. Here, $\sigma \in [0,\overline{\sigma}]$, can be interpreted as a measure of the signal precision[1]. Let $C(\sigma)$ be the cost of acquiring information about the asset quality through by procuring an inspection, with $C^{'}(\sigma)>0$ and $C^{''}(\sigma)>0$, for $\sigma>0$; $C(0)=C^{'}(0)=0$ and $C^{'}(\overline{\sigma})>1$.

The timing of the game is the following: the seller's type is drawn by Nature at the outset; the seller learns his type and chooses a pricing strategy. After observing the price, the buyer updates beliefs on the asset quality and procures the inspection on quality. The inspection's outcome is realized, then the buyer updates beliefs accordingly and makes a purchase decision.

The buyer starts the game with a prior belief on the asset being of high quality, $\mu_0 = Pr(q=1)$, and updates beliefs according to Bayes rule, after observing the price. Note that separating prices $\left(P^b, P^g\right)$ induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. After observing the price and learning his valuation for the asset, the buyer may decide to procure an inspection on quality. Given a price P and a belief μ, the buyer solves

$$\max_{\sigma\in[0,\overline{\sigma}]} \mu\left(v-P\right) - P\left(1-\mu\right)\left(1-\sigma\right) - C\left(\sigma\right).$$

The assumptions on $C(\cdot)$ ensure the existence of a unique solution to this problem, $\sigma^* = \sigma(P,\mu) < \overline{\sigma}$, defined by $C^{'}(\sigma^*) = P(1-\mu)$. Note that $\sigma^* = \sigma(P,\mu)$ is increasing in the price and decreasing in the buyer's assessment of quality. The buyer then decides whether or not to acquire the information amount $\sigma^* = \sigma(P,\mu)$. Note that he will acquire information if and only if his expected utility, given the optimal amount of information, is nonnegative:

$$\mu\left(v-P\right) - P\left(1-\mu\right)\left(1-\sigma^*\right) - C\left(\sigma^*\right) \geq 0.$$

[1]The results obtained in this paper are robust to more general signal structures that satisfies the following assumptions: $Pr\left(F \mid q=1\right) > Pr\left(F \mid q=0\right)$ and $Pr\left(NF \mid q=1\right) < Pr\left(NF \mid q=0\right)$.

If it is the case, he receives a signal $s \in \{F, NF\}$ and makes a purchase decision, which leads to the potential demand

$$\overline{D}(P, \mu) = 1 - G\left(\frac{\mu P + P(1-\mu)(1-\sigma^*) + C(\sigma^*)}{\mu}\right).$$

We denote by $\pi(\theta, P, \mu)$ the one-period profits of a type-θ seller who sets the price P, inducing beliefs μ:

$$\pi(\theta, P, \mu) = PD(\theta, P, \mu),$$

with $D(\theta, P, \mu) = \overline{D}(P, \mu)[\theta + (1-\theta)(1-\sigma^*)]$ denoting the actual demand.

We define and analyze conditions for the existence of separating equilibria in pure strategies. First we characterize the benchmark case, in which the cost of procuring the inspection on quality is zero and σ is a fixed parameter (the buyer always chooses the maximum amount of information), and show that there is no separating equilibrium. We then show that when allowing the buyer to choose the precision of the signal, by costly acquire information, sellers are able to separate themselves in equilibrium. We finally discuss the implications of time-on-the-market on separating equilibria, in the case that the selling season is composed of two periods and there is one asset on sale over both periods.

3 Separating Equilibrium

A separating equilibrium is a sequential equilibrium at which the buyer can distinguish the good and the bad seller by the different pricing choices they made. Note that separating prices allow the buyer to infer the seller's type, but not the true quality of the asset. Separating prices (P^b, P^g) induce beliefs $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$, whereas pooling prices do not provide any information and the posterior will be the same as the prior, $\mu = \mu_0$. Moreover, off-equilibrium prices $P \neq \{P^b, P^g\}$ are assumed to induce pessimistic beliefs $\mu = b$, to make the existence of the equilibrium easier.

Definition 1. *(Separating Equilibrium)* A separating equilibrium is a pair $\left(P^b, P^g\right)$ such that three conditions hold:

C1. $\pi\left(b, P^b, \mu = b\right) \geq \pi\left(b, P, \mu = b\right)$, for every $P \neq P^g$.

C2. $\pi\left(b, P^b, \mu = b\right) \geq \pi\left(b, P^g, \mu = g\right)$, and

C3. $\pi\left(g, P^g, \mu = g\right) \geq \pi\left(g, P, \mu = b\right)$, for every $P \neq P^g$.

For the bad seller, P^b must dominate any price $P \neq P^g$ under pessimistic beliefs (C1). Moreover, the bad seller should not have incentives to mimic the good one, even if this implies optimistic beliefs (C2). For the good seller, P^g must dominate any other price P that induce pessimistic beliefs (C3).

Lemma 2. In any separating equilibrium $P^b = P^{b*}$, where P^{b*} is the maximizer of $\pi\left(b, P, \mu = b\right)$. Moreover, for the good seller, it is sufficient to check that $\pi\left(g, P^g, \mu = g\right) \geq \pi\left(g, P^{g*}, \mu = b\right)$, where P^{g*} is the maximizer of $\pi\left(g, P, \mu = b\right)$.

Proof. $P^b = P^{b*}$ is a necessary condition for C1 to be satisfied. Moreover C3 requires that the good seller should not have any incentive to deviate from the equilibrium price, with such deviation implying pessimistic beliefs. Then it is sufficient to control for best deviation which occurs at P^{g*}, the maximizer of $\pi\left(g, P, \mu = b\right)$. ■

3.1 Benchmark case: costless inspections

In subsequent sections we assume that the buyer chooses the precision of the signal by acquiring information at rising cost. Prior to study this problem, it is useful to have a benchmark case against which to compare the effect of information acquisition on the existence and characterization of a separating equilibrium. Hence in this section the cost of procuring the inspection on quality is zero and $\sigma = \overline{\sigma}$ is a fixed parameter (the buyer always chooses the maximum amount of information). We denote by μ^s the updated belief on $q = 1$, after observing the price and the inspection outcome. Note that the buyer will anticipate procuring a favorable inspection when updating beliefs, so that we can restrict attention to beliefs $\mu^F(\mu)$ since no buyer will buy the asset at any price if $s = NF$. Then $\mu^F(\mu) = \frac{\mu}{\mu+(1-\mu)(1-\overline{\sigma})}$, and $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$. Conditional on a favorable inspection outcome, the buyer will buy the good if $\mu^F(\mu)v - P \geq 0$, which leads to the potential demand $\overline{D}\left(P, \mu^F(\mu)\right) = 1 - G\left(\frac{P}{\mu^F(\mu)}\right)$ and associated profits

$$\pi\left(\theta, P, \mu^F(\mu)\right) = PD\left(\theta, P, \mu^F(\mu)\right),$$

with $D\left(\theta, P, \mu^F(\mu)\right) = \overline{D}\left(P, \mu^F(\mu)\right)\left[\theta + (1-\theta)(1-\overline{\sigma})\right]$ denoting the actual demand.

Lemma 3. With costless inspections, there is no separating equilibrium.

Proof. The pair $\left(P^{b*}, P^g\right)$ is a separating equilibrium if two conditions simultaneously hold:

1. $\pi\left(b, P^{b*}, \mu^F(b)\right) \geq \pi\left(b, P^g, \mu^F(g)\right)$, and

2. $\pi\left(g, P^g, \mu^F(g)\right) \geq \pi\left(g, P^{g*}, \mu^F(b)\right)$.

Consider the equilibrium price $P^g = \overline{P} > P^{b*}$ such that $\pi\left(b, P^{b*}, \mu^F(b)\right) = \pi\left(b, \overline{P}, \mu^F(g)\right)$. At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\pi\left(g, \overline{P}, \mu^F(g)\right) > \pi\left(g, P^{g*}, \mu^F(b)\right)$, which is equivalent to require:

$$\pi\left(g, P^{b*}, \mu^F(b)\right) > \pi\left(g, P^{g*}, \mu^F(b)\right)$$

given that we define $\overline{P}$ such that $\pi\left(b, P^{b*}, \mu^F(b)\right) = \pi\left(b, \overline{P}, \mu^F(g)\right)$. This cannot be true since $P^{g*} = P^{b*} = \frac{\mu^F(b)\left(1-G\left(\frac{P}{\mu^F(b)}\right)\right)}{g\left(\frac{P}{\mu^F(b)}\right)}$. Therefore, the only existing equilibrium of this game is a pooling one. ■

If inspections are free, there is no separating equilibrium mainly because the consumer will always fully inspect both types ($\sigma = \overline{\sigma}$) and the bad type is unable to inhibit information acquisition by lowering the price (as is shown in the previous section, σ is inversely related to price, and if a product is cheap, it is better to buy directly rather than pay for an inspection). Mimicking the good type is therefore the only recourse for the bad type. When inspections have a cost, on the other hand, a separating equilibrium can be achieved because σ varies positively with price (and higher prices imply higher inspection intensity), therefore the bad type has incentive to charge less in order to avoid inspections and revelation of his type.

4 Costly information acquisition

4.1 Separating equilibrium with costly information acquisition

We now consider costly information acquisition. It is useful to note, again, the amount of information acquired by the consumer depends positively on prices and negatively on beliefs. In this context, the price of the asset serves as a learning mechanism for quality via two channels: first, directly as a standard price signal and secondly, as a factor which can encourage or discourage inspections. Moreover, both channels operate in the same direction: high prices signal high quality both via the standard signaling mechanism, but also because a high price is essentially an invitation to inspect.

Suppose now that the buyer can acquire information by choosing the precision of the signal, $\sigma \in [0, \overline{\sigma}]$ at a cost $C(\sigma)$. We analyze conditions for the existence of a separating equilibrium in the one-shot game, in which σ is endogenously determined. Consider a buyer, whose "taste" for the asset is given by v, with beliefs μ about $q = 1$ after observing the equilibrium price and prior to procuring the inspection. The problem facing such a buyer is:

$$\max_{\sigma \in [0, \overline{\sigma}]} \mu (v - P) - P (1 - \mu) (1 - \sigma) - C (\sigma) .$$

The assumptions on $C(\cdot)$ ensure the existence of a unique solution to this problem, $\sigma^* = \sigma(P, \mu) < \overline{\sigma}$, defined by $C'(\sigma^*) = P(1 - \mu)$. Note that $\sigma^* = \sigma(P, \mu)$ is increasing in the price and decreasing in his assessment of quality. Then the buyer will drop out of the market once the price reaches P defined by

$$\mu (v - P) - P (1 - \mu) (1 - \sigma^*) - C (\sigma^*) = 0$$

which leads to a potential demand

$$\overline{D} (P, \mu) = 1 - G \left(\frac{\mu P + P (1 - \mu) (1 - \sigma^*) + C (\sigma^*)}{\mu} \right)$$

and associated profits

$$\pi (\theta, P, \mu) = P D (\theta, P, \mu)$$

$$D (\theta, P, \mu) = \overline{D} (P, \mu) [\theta + (1 - \theta) (1 - \sigma^*)] .$$

A separating equilibrium in defined according to Definition 1.

Lemma 4. In any separating equilibrium, $P^b = P^{b*}$, where P^{b*} is the maximizer of $\pi (b, P, \mu = b)$. For the good seller, it is sufficient to check that $\pi (g, P^g, \mu = g) \geq \pi (g, P^{g*}, \mu = b)$, where P^{g*} is the maximizer of $\pi (g, P, \mu = b)$. Moreover $P^{g*} > P^{b*}$ if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$, where $\frac{\sigma_P}{\sigma}$ represents the price-elasticity of information acquisition and $-\frac{\overline{D}_P}{\overline{D}}$ represents the price-elasticity of demand.

Proof. $P^b = P^{b*}$is a necessary condition for C1 in Definition 1 to be satisfied. The price P^g is a separating equilibrium if it dominates any other price P that induce beliefs $\mu = b$ (C3). Thus it is sufficient to control for the best deviation, which occurs at P^{g*},the price that maximizes profits under the worst belief. We show now that $P^{g*} > P^{b*}$, which is implied by $\frac{\partial^2 \pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$:

$$\frac{\partial^2 \pi (\theta, P, \mu)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta} \{PD_P (\theta, P, \mu) + D (\theta, P, \mu)\}$$

$$= PD_{P\theta} (\theta, P, \mu) + D_\theta (\theta, P, \mu),$$

which is positive if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$. Note that $D_\theta (\theta, P, \mu) = \overline{D} (P, \mu) \sigma (P, \mu) > 0$ and $D_{P\theta} (\theta, P, \mu) = \overline{D}_P (P, \mu) \sigma (P, \mu) + \overline{D} (P, \mu) \sigma_P (P, \mu)$ is positive if $\frac{\sigma_P}{\sigma} \geq -\frac{\overline{D}_P}{\overline{D}}$.■

Proposition 5. There is always a separating equilibrium $\left(P^{b*}, P^g\right)$ with $P^g > P^{g*}$ if $-\frac{\sigma_\mu}{\sigma} \leq \frac{D_\mu}{D}$, where $\frac{\sigma_\mu}{\sigma}$ represents the beliefs-elasticity of information acquisition and $\frac{D_\mu}{D}$ the beliefs-elasticity of demand.

Proof. Consider the price $\overline{P} > P^{b*}$ such that $\pi\left(b, P^{b*}, \mu = b\right) = \pi\left(b, \overline{P}, \mu = g\right)$. If $\overline{P} \leq P^{g*}$, then, by the Envelope Theorem, it is straightforward to show that $\pi\left(g, \overline{P}, \mu = g\right) \geq \pi (g, P^{g*}, \mu = b)$, and $\left(P^{b*}, P^{g*}\right)$ is a separating equilibrium.

Thus we can restrict attention to the case $\overline{P} > P^{g*}$. At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\pi\left(g, \overline{P}, \mu = g\right) \geq \pi (g, P^{g*}, \mu = b)$, which is equivalent to:

$$\pi\left(g, \overline{P}, \mu = g\right) - \pi (g, P^{g*}, \mu = b) \geq \pi\left(b, \overline{P}, \mu = g\right) - \pi\left(b, P^{b*}, \mu = b\right) = 0.$$

We can rewrite the left and right hand sides of this inequality as

$$\left[\pi\left(g, \overline{P}, \mu = g\right) - \pi\left(g, \overline{P}, \mu = b\right)\right] + \left[\pi\left(g, \overline{P}, \mu = b\right) - \pi (g, P^{g*}, \mu = b)\right] \geq$$

$$\left[\pi\left(b,\overline{P},\mu=g\right)-\pi\left(b,\overline{P},\mu=b\right)\right]+\left[\pi\left(b,\overline{P},\mu=b\right)-\pi\left(b,P^{b*},\mu=b\right)\right].$$

Therefore it is enough to show that

1. $\left[\pi\left(g,\overline{P},\mu=g\right)-\pi\left(g,\overline{P},\mu=b\right)\right]\geq\left[\pi\left(b,\overline{P},\mu=g\right)-\pi\left(b,\overline{P},\mu=b\right)\right]$ and

2. $\left[\pi\left(g,\overline{P},\mu=b\right)-\pi\left(g,P^{g*},\mu=b\right)\right]\geq\left[\pi\left(b,\overline{P},\mu=b\right)-\pi\left(b,P^{b*},\mu=b\right)\right]$.

Condition 1 is implied by $\frac{\partial^2\pi(\theta,P,\mu)}{\partial\theta\partial\mu}>0$:

$$\frac{\partial^2\pi\left(\theta,P,\mu\right)}{\partial\theta\partial\mu}=\frac{\partial}{\partial\mu}\left\{PD_\theta\left(\theta,P,\mu\right)\right\}$$

$$=\overline{D}_\mu\left(P,\mu\right)\sigma\left(P,\mu\right)+\overline{D}\left(P,\mu\right)\sigma_\mu\left(P,\mu\right)$$

which is positive if $-\frac{\sigma_\mu}{\sigma}\leq\frac{D_\mu}{D}$.

Since $P^{g*}>P^{b*}$condition 2 is implied by $\frac{\partial^2\pi(\theta,P,\mu)}{\partial P\partial\theta}>0$, which is true if $\frac{\sigma_P}{\sigma}\geq-\frac{\overline{D}_P}{\overline{D}}$, as shown in Lemma 4.■

Two conditions therefore guarantee the existence of a separating equibrium in which high prices signal high quality: $\frac{\partial^2\pi(\theta,P,\mu)}{\partial P\partial\theta}>0$ and $\frac{\partial^2\pi(\theta,P,\mu)}{\partial\theta\partial\mu}>0$. The first condition is standard single-crossing: the cost of signaling through high prices is lower for the good type. The second condition requires that the shift from pessimistic to optimistic beliefs is more attractive to the good type than to his bad-type counterpart. In our setup, signaling requires profits to be marginally more sensitive to type for both prices P and beliefs μ. Note that this is in contrast to Spence´s job-market signaling model where the worker´s utility is quasilinear in beliefs, therefore the second condition is automatically satisfied (with equality). Figure 1 illustrates the role of the above-mentioned conditions for the proof.

The separating equilibrium for the two-period extension of the model can be found in the

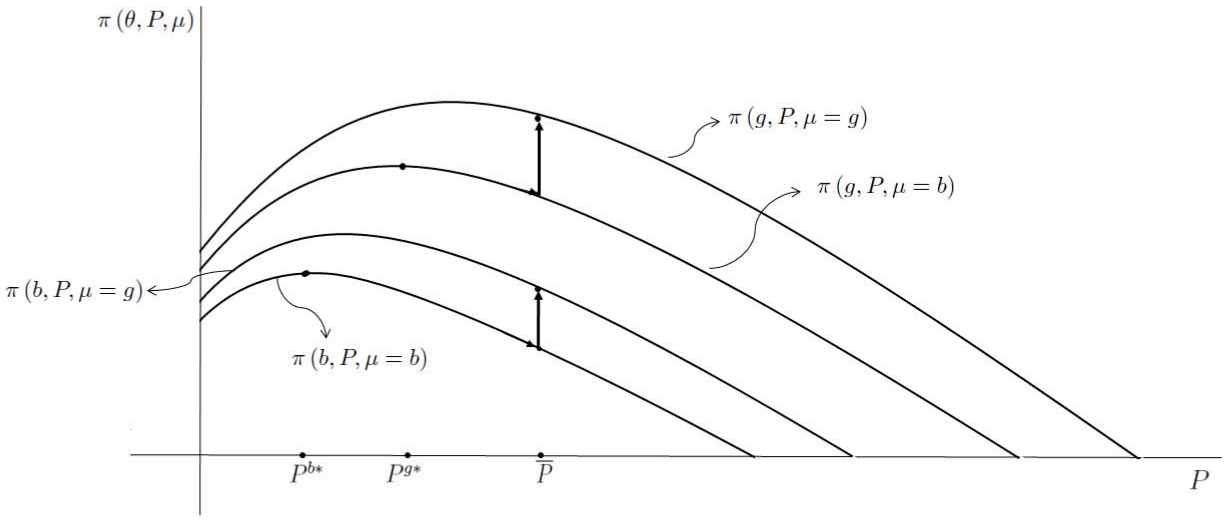

Figure 1:

appendix.

Most of these separating equilibria involve beliefs that are implausible, since any deviation is interpreted as coming from a bad seller. To solve the problem of multiplicity of equilibria that arises in standard signalling games, we restrict attention only to those equilibria that satisfy the "intuitive criterion" of Cho and Kreps (1987). To understand the Cho and Kreps refinement in this context, consider an equilibrium in which the good monopolist's profits are $\pi(g, P^g, \mu = g)$ while the bad firm earns profits $\pi\left(b, P^{b*}, \mu = b\right)$. The equilibrium fails the intuitive criterion if there exists a price P' such that: a) $\pi\left(g, P', \mu = g\right) \geq \pi(g, P^g, \mu = g)$ and b) $\pi\left(b, P', \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. That is, if there exists a price P' such that the good seller is better off by deviating and the bad one makes more profits following the equilibrium price, even if the deviation would have generated optimistic beliefs. Intuitively, if such a price P' exists, consumers should interpret such a deviation as coming from a good seller, making the equilibrium fail.

Proposition 6. The pair $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion.

Proof. The proof consists of two steps. We first show that there is no equilibrium price $P > \overline{P}$

that satisfies the intuitive criterion. Consider the price $P > \overline{P}$ such that $\left(P^{b*}, \overline{P}\right)$ is a separating equilibrium. Define $P^{'} = P - \varepsilon$. Then it is easy to see that a) $\pi\left(g, P^{'}, \mu = g\right) \geq \pi\left(g, P^{g}, \mu = g\right)$ and b) $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. Let P^{g**} be the price that maximizes the good firm's profits under the most optimistic beliefs $\mu = g$, $P^{g**} = argmax_{P}\, \pi\left(g, P, \mu = g\right)$. Noting that $P^{g*} < P^{g**}$ and $P^{g*} < \overline{P} < P^{'}$, we get that $P^{g**} < P^{'} < P$. Therefore $\pi\left(g, P^{'}, \mu = g\right) \geq \pi\left(g, P^{g}, \mu = g\right)$. By Proposition 5 we know that $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$ to ensure that the bad seller would not deviate, then by continuity $\pi\left(b, P, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. Thus for any price $P > \overline{P}$ condition a) is not satisfied, violating the intuitive criterion. We now show that $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion, that is there is no equilibrium price $P^{'}$ such that a) $\pi\left(g, P^{'}, \mu = g\right) \geq \pi\left(g, P^{g}, \mu = g\right)$ and b) $\pi\left(b, P^{'}, \mu = g\right) < \pi\left(b, P^{b*}, \mu = b\right)$. If $P^{'} > \overline{P}$ condition a) is not satisfied. Then, $P^{'} \leq \overline{P}$. But if $P^{'} < \overline{P}$, there is no separating equilibrium, as shown in Proposition 5. Then it must be $P^{'} = \overline{P}$, and $\left(P^{b*}, \overline{P}\right)$ is the only separating equilibrium that satisfies the intuitive criterion. ■

4.2 Comparative static on the equilibrium price $\overline{P}$

In the last section we applied the "intuitive criterion" of Cho and Kreps (1987) to select among equilibria in order to rule out counterintuitive equilibria driven by pessimistic beliefs and we show that the only equilibrium that satisfies the intuitive criterion is given by the pair $\left(P^{b*}, \overline{P}\right)$. In the separating equilibrium the seller's type is perfectly inferred by consumers. The bad-type seller acts as in a perfect information environment, maximizing profits given pessimistic beliefs, whereas the good type has incentives to separate at prices that are not profitable for the bad-seller to imitate even if they generate optimistic beliefs. The only price that survives the intuitive criterion refinement is the less-costly for the good seller. We now analyze how does the price $\overline{P}$ vary with the seller's type θ and with the cost of acquiring information $C\left(\sigma\right)$.

The separating equilibrium price $\overline{P}$ is increasing in $\theta = g$, whereas it can be either increasing or decreasing in $\theta = b$.

Proof. We now show that the price $\overline{P}$ charged by a good seller in the only separating equilibrium that satisfies the intuitive criterion is increasing in his type $\theta = g$. As we assumed for the proof of Proposition 5, the price $\overline{P} > P^{b*}$ is such that $\pi\left(\theta = b, P^{b*}, \mu = b\right) = \pi\left(\theta = b, \overline{P}, \mu = g\right)$. Thus we need to study how the last equality varies with $\theta = g$:

$$\frac{\partial \pi\left(\theta = b, P^{b*}, \mu = b\right)}{\partial \theta} \mid_{\theta=g} = \frac{\partial \pi\left(\theta = b, \overline{P}, \mu = g\right)}{\partial \theta} \mid_{\theta=g}$$

$$\frac{\partial \pi\left(\theta = b, P^{b*}, \mu = b\right)}{\partial \theta} \mid_{\theta=g} + \frac{\partial \pi\left(\theta = b, P^{b*}, \mu = b\right)}{\partial P^{b*}} \frac{\partial P^{b*}}{\partial \theta} \mid_{\theta=g} + \frac{\partial \pi\left(\theta = b, P^{b*}, \mu = b\right)}{\partial \mu} \mid_{\mu=g}$$

$$= \frac{\partial \pi\left(\theta = b, \overline{P}, \mu = g\right)}{\partial \theta} \mid_{\theta=g} + \frac{\partial \pi\left(\theta = b, \overline{P}, \mu = g\right)}{\partial \overline{P}} \frac{\partial \overline{P}}{\partial \theta} \mid_{\theta=g} + \frac{\partial \pi\left(\theta = b, \overline{P}, \mu = g\right)}{\partial \mu} \mid_{\mu=g} .$$

Note that the derivatives on the left-hand side of the equality are cero, therefore:
$0 = \frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \overline{P}} \frac{\partial \overline{P}}{\partial \theta} \mid_{\theta=g} + \frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \mu}$. Then we need to show that $\frac{\partial \overline{P}}{\partial \theta} \mid_{\theta=g} > 0$:

$$\frac{\partial \overline{P}}{\partial \theta} \mid_{\theta=g} = -\frac{\frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \mu}}{\frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \overline{P}}} > 0,$$

which is indeed positive since $\frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \mu} > 0$ and $\frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \overline{P}} < 0$ (the marginal income of the monopolist corresponding to the equilibrium price $\overline{P}$ is negative).

The same argument applies when analyzing the effect of a variation of $\theta = b$ on the equilibrium price $\overline{P}$. We now show that a change in $\theta = b$ affects ambigously the equilibrium price $\overline{P}$. Again we need to study how the equality $\pi\left(\theta = b, P^{b*}, \mu = b\right) = \pi\left(\theta = b, \overline{P}, \mu = g\right)$ varies with $\theta = b$. Then:

$$\frac{\partial \overline{P}}{\partial \theta} \mid_{\theta=b} = \frac{\left[\frac{\partial \pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial \theta} - \frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \theta}\right] + \frac{\partial \pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial \mu}}{\frac{\partial \pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial \overline{P}}}.$$

Note that $\left[\frac{\partial\pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial\theta}-\frac{\partial\pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial\theta}\right]<0$ is implied by the single-crossing conditions $\frac{\partial^2\pi(\theta,P,\mu)}{\partial P\partial\theta}>0$ and $\frac{\partial^2\pi(\theta,P,\mu)}{\partial\theta\partial\mu}>0$ discussed in the proof of Proposition 5. Moreover we know that $\frac{\partial\pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial\mu}>0$ and $\frac{\partial\pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial\overline{P}}<0$. Then depending on whether $\mid\frac{\partial\pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial\theta}-\frac{\partial\pi\left(\theta=b,\overline{P},\mu=g\right)}{\partial\theta}\mid$ is greater or smaller than $\frac{\partial\pi\left(\theta=b,P^{b*},\mu=b\right)}{\partial\mu}$, the price $\overline{P}$ may increase or decrease in $\theta=b$, respectively.

We now consider a specific funciton for the cost of acquiring information in order to study how it affects the equilibrium price. In particular let $C(\sigma,\lambda)=\frac{\lambda\sigma^2}{2}$ be the cost of acquiring information about the asset quality, with $C'(\sigma,\lambda)\equiv C_\sigma>0$, $C_{\sigma\sigma}>0$, $C_\lambda>0$ and $C_{\sigma\lambda}>0$.

The separating equilibrium price $\overline{P}$ may be either increasing or decreasing in the cost of acquiring information λ.

Proof. Let $\pi(\theta,P(\lambda),\mu)$ be the profit function of a type-θ seller who charges the price P inducing beliefs μ, with

$$\pi(\theta,P(\lambda),\mu)=P(\lambda)\,\overline{D}(P(\lambda),\lambda,\mu)\left[\theta+(1-\theta)(1-\sigma^*(P(\lambda),\lambda,\mu))\right].$$

We need to analyze the effect of the parameter λ on both sides of the following equality:

$$\pi\left(\theta=b,P^{b*}(\lambda),\mu=b\right)=\pi\left(\theta=b,\overline{P}(\lambda),\mu=g\right).$$

Then:

$$\frac{\partial\overline{P}}{\partial\lambda}=\frac{\left[P^{b*}(\lambda)\frac{\partial\overline{D}\left(P^{b*}(\lambda),\lambda,\mu=b\right)}{\partial\lambda}\left[b+(1-b)\left(1-\sigma^*\left(P^{b*}(\lambda),\lambda,\mu=b\right)\right)\right]-P^{b*}(\lambda)\,\overline{D}\left(P^{b*}(\lambda),\lambda,\mu=b\right)(1-b)\frac{\partial\sigma^*\left(P^{b*}(\lambda),\lambda,\mu=b\right)}{\partial\lambda}\right]}{\frac{\partial\pi\left(\theta=b,\overline{P}(\lambda),\mu=g\right)}{\overline{P}(\lambda)}}$$

$$-\frac{\left[\overline{P}(\lambda)\frac{\partial\overline{D}\left(\overline{P}(\lambda),\lambda,\mu=g\right)}{\partial\lambda}\left[b+(1-b)\left(1-\sigma^*\left(\overline{P}(\lambda),\lambda,\mu=g\right)\right)\right]-\overline{P}(\lambda)\,\overline{D}\left(\overline{P}(\lambda),\lambda,\mu=g\right)(1-b)\frac{\partial\sigma^*\left(\overline{P}(\lambda),\lambda,\mu=g\right)}{\partial\lambda}\right]}{\frac{\partial\pi\left(\theta=b,\overline{P}(\lambda),\mu=g\right)}{\overline{P}(\lambda)}}.$$

Note that the effect of an increase in the cost of acquiring information is negative both on the potential demand $\overline{D}(P(\lambda),\lambda,\mu)$ and the optimal amount of information acquisition $\sigma^*(P(\lambda),\lambda,\mu)$. Nevertheless the reduction in the potential demand and the optimal amount of acquired information is marginally lower in correspondence of higher beliefs, whereas it is marginally higher when the price charged is higher. Therefore the overall impact of an increase in the cost of acquiring information is ambiguous.

5 Time-on-the-Market

5.1 Costly information acquisition and time-on-the-market

We now consider a selling season composed of two periods with only one asset on sale over both periods. Note that now both price and time-on-the-market may signal quality. In this case the game proceeds as follows: at the beginning of the first period the seller posts a separating price P. After observing the price, the buyer updates beliefs on the asset quality and procures an inspection on quality, then decides whether to buy or not. If no sale occurs in the first period, then we get to the second stage of the game, where a new buyer enters the market. The second-period buyer takes into account the fact that the asset did not sell in the first period at price P (time-on-the-market) when forming beliefs. He then procures an inspection on quality, and makes a purchase decision. Note that he cannot observe the outcome of the first-period inspection or even if one was procured, therefore he did not know why the asset did not sell in the first period. Specifically, the buyer cannot distinguish between two possible reasons: (i) the asset was overpriced with respect to first-period buyer valuation, or (ii) the inspection's outcome was unfavorable revealing low quality, even if the first buyer was ready to buy it. Therefore, time-on-the-market enters as a variable at the time of updating beliefs on the seller's type and the quality of the asset.

We solve the game by backward induction. The problem facing the buyer in $t = 2$ is

$$\max_{\sigma_2 \in [0,\overline{\sigma}]} \mu_2 (v - P_2) - P_2 (1 - \mu_2) (1 - \sigma_2) - C (\sigma_2),$$

which leads to $\sigma_2^* = \sigma (P_2, \mu_2)$ such that $C' (\sigma_2^*) = P_2 (1 - \mu_2)$, where μ_2 denotes the beliefs about the asset being of high quality, taking into account time-on-the-market:

$$\mu_2(P,\mu) = \frac{\mu G(f(P,\mu))}{G(f(P,\mu)) + [1 - G(f(P,\mu))]\,\sigma(P,\mu)(1-\mu)},$$

where $G(f(P,\mu))$ denotes the probability that second-period buyer assigns to reason (i):

$$G(f(P,\mu)) = G\left(\frac{\mu P + (1-\mu)(1-\sigma^*(P,\mu))P + C(\sigma^*(P,\mu))}{\mu}\right)$$

and $[1 - G(f(P,\mu))]\,\sigma(P,\mu)(1-\mu)$ denotes the probability that a flaw was discovered at the inspection (reason (ii)).

Note that, when time-on-the-market comes into play, not only the buyer updates beliefs accordingly, but also the seller. We denote by $\widetilde{\theta}$ the posterior probability assigned by the seller to high quality given that asset did not sell in the previous period:

$$\widetilde{\theta}(\theta,P,\mu) = \frac{\theta G(f(P,\mu))}{G(f(P,\mu)) + [1 - G(f(P,\mu))]\,\sigma(P,\mu)(1-\theta)}.$$

Hence, a buyer with valuation v for the asset will drop out of the market once the price reached P_2 defined by

$$\mu_2(v - P_2) - P_2(1-\mu_2)(1-\sigma_2^*) - C(\sigma_2^*) = 0$$

which leads to second-period demand

$$\overline{D}_2(P_2,\mu_2) = 1 - G\left(\frac{\mu_2 P_2 + P_2(1-\mu_2)(1-\sigma_2^*) + C(\sigma_2^*)}{\mu_2}\right),$$

and associated profits

$$\pi\left(\widetilde{\theta},P_2,\mu_2\right) = P_2 D_2\left(\widetilde{\theta},P_2,\mu_2\right) = P_2\overline{D}_2(P_2,\mu_2)\left[\widetilde{\theta} + \left(1-\widetilde{\theta}\right)(1-\sigma_2^*)\right].$$

Note that the inspection procured by the buyer in the second period is more intense than the one procured by first-period "potential" buyer, $\sigma_2^* \geq \sigma_1^*$. As $\mu_2 < \mu$, the result follows from $\frac{\partial\sigma^*}{\partial\mu} = -\frac{P}{C''(\sigma^*)} < 0$. This is because second-period buyer worries that time-on-the-market was due to an unfavorable inspection, therefore detection of low quality, in the first period and as a result procures a more intense inspection in the second period.

For any history of separation $\left(P^{b*}, P^{g}\right)$, second-period prices are given by $\left(P_2^{b*}, P_2^{g**}\right)$, where P_2^{b*} is the maximizer of $\pi\left(\widetilde{b}, P_2, \mu_2\left(P^{b*}, \mu = b\right)\right)$ and P_2^{g**} is the maximizer of $\pi\left(\widetilde{g}, P_2, \mu_2\left(P^{g}, \mu = g\right)\right)$. We denote by $\Pi\left(\theta, P, \mu\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs μ, and look for separating equilibria as well.

When there is only one asset on sale (therefore both price and time-on-the-market can signal quality) there might be no separating equilibrium even if single-crossing is satisfied. The key to this result is that second-period buyer cannot observe why the asset did not sell in the first period. Notably, the failure to sell can be attributed to (i) overpricing or (ii) an unfavorable inspection outcome. Therefore the good seller can influence buyer beliefs by choosing a high price, in order to make reason (i) seem more plausible. Failure to sell in the first period conveys a much weaker assessment of quality when the price is high than when it is low. The problem, however, is that the bad seller has more incentive to hide behind a high initial price because he benefits more from an increase in buyer beliefs than his high-quality counterpart, given his lower probability of sale in the first period. It follows that the bad type will always prefer to imitate a high quality seller using a high initial price because the change in consumer beliefs that results from a first-period sale inflict more damage than a non-sale, especially if the latter can be easily justified by a high price.

Fact 7. With information acquisition and time-on-the-market on of the sufficient condition for the existence of a separating equilibrium, $\frac{\partial^2 \Pi(\theta, P, \mu)}{\partial \theta \partial \mu} > 0$ (1) is not satisfied.

Even if we can assure single-crossing, condition (1) cannot be satisfied, since the shift from pessimistic to optimistic beliefs is more attractive to the bad seller, making the equilibrium fail:

$$\frac{\partial^2 \Pi\left(\theta, P, \mu\right)}{\partial \theta \partial \mu} = \frac{\partial^2}{\partial \theta \partial \mu} \left\{ \left[P - \pi\left(\widetilde{\theta}, P_2, \mu_2\right)\right] D\left(\theta, P, \mu\right) + \pi\left(\widetilde{\theta}, P_2, \mu_2\right) \right\}$$

$$= \frac{\partial}{\partial \mu} \left\{ \left[P - \pi \left(\widetilde{\theta}, P_2, \mu_2 \right) \right] D_\theta (\theta, P, \mu) + [1 - D (\theta, P, \mu)] \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta} \right\}$$

$$= \left[P - \pi \left(\widetilde{\theta}, P_2, \mu_2 \right) \right] D_{\theta\mu} (\theta, P, \mu) - \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \mu} D_\theta (\theta, P, \mu)$$

$$+ [1 - D (\theta, P, \mu)] \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta \partial \mu} - \frac{\partial \pi \left(\widetilde{\theta}, P_2, \mu_2 \right)}{\partial \theta} D_\mu (\theta, P, \mu) .$$

There are two terms in the previous expression which are unequivocally negative, and possibly quite important. The first is $\left[-\frac{\partial \pi(\widetilde{\theta}, P_2, \mu_2)}{\partial \mu} D_\theta (\theta, P, \mu) \right]$. This reflects the fact that inducing better beliefs at $t = 2$ (higher μ_2 through optimistic μ) is more valuable for the bad type, since he is more likely to have an unsold asset at that time. The second term, $\left[-\frac{\partial \pi(\widetilde{\theta}, P_2, \mu_2)}{\partial \theta} D_\mu (\theta, P, \mu) \right]$, shows that inducing higher first-period beliefs is more appealing to the bad type than the good, since the outside option of waiting until the next period is worse for the former.

5.2 Costly information acquisition, time-on-the-market and separating equilibrium

We now consider the case in which only first-period buyer is allowed to acquire information to infer the common value of the object, whereas the second-period one learns from his predecessor purchase decision and makes his choice accordingly. We show there is a separating equilibrium in which high prices (and time-on-the-market) signal high quality. Note that now the second-period buyer updates beliefs about the asset quality only observing that no sale occurred in the first period at price P.

We solve the game by backward induction. Since no inspection on quality occurs in the second period, given beliefs μ_2 , the buyer will buy if $\mu_2 v - P_2 \geq 0$, which leads to second-period demand

$$\overline{D}_2 (P_2, \mu_2) = 1 - G \left(\frac{P_2}{\mu_2} \right),$$

and associated profits

$$\pi\left(P_2,\mu_2\right)=P_2\overline{D}_2\left(P_2,\mu_2\right),$$

where μ_2 is defined as before and denotes the beliefs about the asset being of high quality, taking into account time-on-the-market:

$$\mu_2\left(P,\mu\right)=\frac{\mu G\left(f\left(P,\mu\right)\right)}{G\left(f\left(P,\mu\right)\right)+\left(1-G\left(f\left(P,\mu\right)\right)\right)\left(\sigma\left(P,\mu\right)\left(1-\mu\right)\right)}.$$

For any history of separation $\left(P^{b*},P^{g}\right)$, second-period prices are given by $\left(P_2^{b*},P_2^{g**}\right)$, where P_2^{b*} is the maximizer of $\pi\left(P_2,\mu_2\left(P^{b*},\mu=b\right)\right)$ and P_2^{g**} is the maximizer of $\pi\left(P_2,\mu_2\left(P^{g},\mu=g\right)\right)$. We denote by $\Pi\left(\theta,P,\mu\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs μ, and look for separating equilibria as well.

Proposition 8. With first-period information acquisition and time-on-the-market there is a separating equilibrium $\left(P^{b*},P^{g}\right)$ with $P^{g}>P^{g*}$, if the following two conditions hold:

1. $\overline{g}\leq\frac{G(P)^2}{\overline{\sigma}}$,

2. $\frac{\partial D_\theta}{\partial\mu}\frac{\mu}{D_\theta}\geq-\frac{\partial[P-\pi(P_2,\mu_2)]}{\partial\mu}\frac{\mu}{[P-\pi(P_2,\mu_2)]}$.

Proof. We can write $\Pi\left(\theta,P,\mu\right)$ as

$$\Pi\left(\theta,P,\mu\right)=\left[P-\pi\left(P_2,\mu_2\right)\right]D\left(\theta,P,\mu\right)+\pi\left(P_2,\mu_2\right),$$

with

$$D\left(\theta,P,\mu\right)=\overline{D}\left(P,\mu\right)\left[\theta+\left(1-\theta\right)\left(1-\sigma\left(P,\mu\right)\right)\right].$$

As we know, we need to check that:

1) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial P \partial \theta} > 0$ and

2) $\frac{\partial^2 \Pi(\theta,P,\mu)}{\partial \theta \partial \mu} > 0$.

Condition (1) is equal to:

$$\frac{\partial^2 \Pi(\theta, P, \mu)}{\partial P \partial \theta} = \frac{\partial}{\partial \theta} \left\{ [P - \pi(P_2, \mu_2)] D_P(\theta, P, \mu) + \left[1 - \frac{\partial \pi(P_2, \mu_2)}{\partial P}\right] D(\theta, P, \mu) + \frac{\partial \pi(P_2, \mu_2)}{\partial P} \right\}$$

$$= [P - \pi(P_2, \mu_2)] D_{P\theta}(\theta, P, \mu) + \left[1 - \frac{\partial \pi(P_2, \mu_2)}{\partial P}\right] D_\theta(\theta, P, \mu),$$

Note that we are looking for separation through high prices, so that we can assume $P > \pi(P_2, \mu_2)$. Moreover $D_{P\theta}(\theta, P, \mu) = \overline{D}_P(P, \mu) \sigma(P, \mu) + \overline{D}(P, \mu) \sigma_P(P, \mu)$ is positive if $\frac{\sigma_P}{\sigma} > -\frac{\overline{D}_P}{\overline{D}}$ (as shown in Lemma 4) and $D_\theta(\theta, P, \mu) = \overline{D}(P, \mu) \sigma(P, \mu) > 0$. So that it is enough to show that $\frac{\partial \pi(P_2,\mu_2)}{\partial P} < 1$:

$$\frac{\partial \pi(P_2, \mu_2)}{\partial P} = \frac{P_2^{*2}}{\mu_2^2} g\left(\frac{P_2^*}{\mu_2}\right) \frac{\partial \mu_2}{\partial P} = \left[1 - G\left(\frac{P_2^*}{\mu_2}\right)\right] \frac{1 - G\left(\frac{P_2^*}{\mu_2}\right)}{g\left(\frac{P_2^*}{\mu_2}\right)} \frac{\partial \mu_2}{\partial P}$$

using the fact that at the optimum $\frac{P_2^*}{\mu_2} = \frac{1-G\left(\frac{P_2^*}{\mu_2}\right)}{g\left(\frac{P_2^*}{\mu_2}\right)}$. So that it is enough to show that $\frac{\partial \mu_2}{\partial P} < 1$:

$$\frac{\partial \mu_2}{\partial P} = \frac{\partial}{\partial P} \left\{ \frac{\mu G(f(P, \mu))}{G(f(P, \mu)) + (1 - G(f(P, \mu)))(\sigma(P, \mu)(1 - \mu))} \right\}$$

$$= \frac{\mu(1 - \mu)\left[g(f)\sigma^* \frac{\partial f(P,\mu)}{\partial P} - \sigma_P^* G(f)(1 - G(f))\right]}{[G(f) + (1 - G(f))(\sigma^*(1 - \mu))]^2}$$

$$= \frac{\mu(1 - \mu)\left[g(f)\sigma^* \left(\frac{1-\sigma^*(1-\mu)}{\mu}\right) - \sigma_P^* G(f)(1 - G(f))\right]}{[G(f) + (1 - G(f))(\sigma^*(1 - \mu))]^2}$$

using the fact that

$$f(P,\mu)=\frac{\mu P+(1-\mu)(1-\sigma^*)P+C(\sigma^*)}{\mu}$$

and

$$\frac{\partial f(P,\mu)}{\partial P}=\frac{\mu+(1-\mu)(1-\sigma^*)}{\mu}.$$

Now if

$$\left[g(f)\sigma^*\left(\frac{1-\sigma^*(1-\mu)}{\mu}\right)-\sigma_P^* G(f)(1-G(f))\right]\leq 0$$

the proof is completed. From now on we consider the case in which

$$\left[g(f)\sigma^*\left(\frac{1-\sigma^*(1-\mu)}{\mu}\right)-\sigma_P^* G(f)(1-G(f))\right]>0.$$

$$\frac{\partial \mu_2}{\partial P}=\frac{\mu(1-\mu)\left[g(f)\sigma^*\left(\frac{1-\sigma^*(1-\mu)}{\mu}\right)-\sigma_P^* G(f)(1-G(f))\right]}{\left[G(f)+(1-G(f))(\sigma^*(1-\mu))\right]^2}<1$$

$$(1-\mu)g(f)\sigma^*-(1-\mu)^2 g(f){\sigma^*}^2-\sigma_P^*\mu(1-\mu)G(f)(1-G(f))$$

$$<G(f)^2+(1-G(f))^2(\sigma^*(1-\mu))^2+2G(f)(1-G(f))(\sigma^*(1-\mu)).$$

Then it is enough to show that

$$(1-\mu)g(f)\sigma^*<G(f)^2,$$

moreover as $(1-\mu)g(f)\sigma^*\leq(1-\mu)g(f)\overline{\sigma}$, and $f(P,\mu)=\frac{\mu P+(1-\mu)(1-\sigma^*)P+C(\sigma^*)}{\mu}>P$ implies $G(f)>G(P)$, it is enough that

$$(1-\mu)g(f)\overline{\sigma}<G(P)^2.$$

Therefore $\frac{\partial\pi(P_2,\mu_2)}{\partial P}<1$ is always true if $\overline{g}\leq\frac{G(P)^2}{\overline{\sigma}}$.
At the same time we require $\frac{\partial^2\Pi(\theta,P,\mu)}{\partial\theta\partial\mu}>0$:

$$\frac{\partial^2 \Pi(\theta, P, \mu)}{\partial\theta\partial\mu} = \frac{\partial}{\partial\mu}\left\{\left[P - \pi(P_2, \mu_2)\right] D_\theta(\theta, P, \mu)\right\} =$$

$$\left[P - \pi(P_2, \mu_2)\right] D_{\theta\mu}(\theta, P, \mu) - \frac{\partial\pi(P_2, \mu_2)}{\partial\mu} D_\theta(\theta, P, \mu) > 0$$

if

$$\frac{\partial D_\theta}{\partial\mu}\frac{\mu}{D_\theta} \geq -\frac{\partial\left[P - \pi(P_2, \mu_2)\right]}{\partial\mu}\frac{\mu}{\left[P - \pi(P_2, \mu_2)\right]}.\blacksquare$$

6 Conclusion

This paper examines the optimal pricing strategy in a monopoly market with asymmetric information about product quality and information acquisition. The seller has private information about the probability of owning a high-quality asset, whereas the buyer is initially uninformed about quality. The buyer has two learning mechanisms: price signaling and information acquisition prior to purchase. Information acquisition is costly - in terms of time and money - and imprecise. Moreover, it is increasing in prices and decreasing in the prior assessment of quality. We show existence of a unique separating equilibrium that satisfies the intuitive criterion in which high prices signal high quality because a high price is essentially an invitation to inspect.

We then discuss the implications of time-on-the-market on the separating equilibrium, when the selling season is composed of two periods and there is only one object for sale. The two conditions that guarantee the existence of a separating equilibrium might not hold in this case, since a failure to sell in the first period conveys a much weaker assessment of quality when the price is high than when it is low. Hence the low-quality seller has more incentive to hide behind a high initial price and will always prefer to imitate a high quality seller. We solve the problem by allowing only first-period buyers to acquire information. When the second-period buyer learns from the price history and his predecessors purchase decision, a separating equilibrium exists in which high prices (and time-on-the-market) signal high quality.

Our model has many applications and can help explain price dynamics in real estate, auto, arts and clothes markets, for example. Consumers of high-end products usually spend more time researching product attributes. In particular, real estate and auto purchase decisions are usually made after inspections that range from casual to professional, and have corresponding costs. We explain the price path observable in such situations.

Many interesting extensions can be derived from this analysis. For example, a finite or infinite horizon may be used to study price dynamics. Furthermore, a multi-period framework may illuminate the unresolved conclusion about the existence of a separating equilibrium when both prices and time-on-the-market signal quality. That open question may also be approached by discounting the future in different ways for the high-quality and low-quality seller, respectively. Finally, the analysis of different market structures, where strategic interaction among firms also comes into play, might yield different conclusions about the optimal pricing strategy.

References

[1] AKERLOF, G. A. (1970): “The Market for "Lemons": Quality Uncertainty and the Market Mechanism”, *The Quarterly Journal of Economics*, Vol. 84, No. 3., pp. 488-500.

[2] BAGWELL, K. and M. RIORDAN (1991): “High and Declining Prices Signal Product Quality”, *The American Economic Review*, Vol. 81, No. 1, pp. 224-239.

[3] BERGEMANN, J. and J. VALIMAKI (1996): “Learning and Strategic Pricing”, *Econometrica*, Vol. 64, pp. 1125-49.

[4] BERGEMANN, J. and J. VALIMAKI (1997): “Market Diffusion with Two-Sided Learning”, *The RAND Journal of Economics*, Vol. 28, No. 4, pp. 773-795.

[5] BERGEMANN, J. and J. VALIMAKI (2005): “Monopoly Pricing of Experience Goods”, Cowles Foundation Discussion Paper No. 1463 .

[6] BESTER, H. and K. RITZBERGER (2001): “Strategic pricing, signalling, and costly information acquisition”, *International Journal of Industrial Organization,* Vol. 19, pp. 1347-1361.

[7] CHADE, H. and V. VERA de SERIO (2002): “Pricing, learning, and strategic behavior in a single-sale model”, *Economict Theory,* Vol. 19, pp. 333-353.

[8] CHAN, Y. and H. LELAND (1982): “Prices and Qualities in Markets with Costly Information”, *The Review of Economic Studies*, Vol. 49, No. 4, pp. 499-516.

[9] CHO, I. and D. KREPS (1987): “Signaling Games and Stable Equilibria”, *The Quarterly Journal of Economics*, Vol. 102, No. 2, pp. 179-221.

[10] GERTZ, C. (2014): “Quality Uncertainty with Imperfect Information Acquisition”, Institute of Mathematical Economics, Working paper 487.

[11] JUDD, K. and M. RIORDAN (1994): “Price and Quality in a New Product Monopoly”, *The Review of Economic Studies,* Vol. 61, No. 4, pp. 773-789.

[12] KIHLSTROM, R. (1974): “A Bayesian Model of Demand for Information about Product Quality”, *International Economic Review,* Vol. 15, No. 1, pp. 99-118..

[13] LINNEMER, L. (2002): “Price and Advertising as Signals of Quality when Some Consumers are Informed”, *International Journal of Industrial Organization,* Vol. 20, pp. 931-947.

[14] MEZZETTI, C. and T. TSOULOUHAS (2000): “Gathering information before signing a contract with a privately informed principal”, *International Journal of Industrial Organization,* Vol. 18, pp. 667-689.

[15] MILGROM, P. (1981): “Rational Expectations, Information Acquisition, and Competitive Bidding”, *Econometrica,* Vol. 49, No. 4, pp. 921-943.

[16] MILGROM, P. and J. ROBERTS (1986): “Price and Advertising Signals of Product Quality”, *Journal of Political Economy,* Vol. 94, No. 4, pp. 796-821.

[17] SCHLEE, E. (2001): "Buyer Experimentation and Introductory Pricing", *Journal of Economic Behavior and Organization,* Vol. 44, pp. 347-362.

[18] TAYLOR, C. (1999): "Time-on-the-Market as a Sign of Quality", *The Review of Economic Studies,* Vol. 66, pp. 555-578.

[19] VOORNEVELD, M. and J.W. WEIBULL (2011): "A Scent of Lemon—Seller Meets Buyer with a Noisy Quality Observation", *Games*, Vol. 2, pp. 163-186.

[20] WOLINSKY, A. (1983): "Prices as Signals of Product Quality", *The Review of Economic Studies*, Vol. 50, No. 4, pp. 647-658.

Appendix

Costless inspections and two-period game

We now consider the selling season is composed of two periods and there is an object for sale each period. We denote by μ^s the updated belief on $q = 1$, after observing the price and the inspection outcome. We restrict attention to beliefs $\mu^F(\mu)$ since no buyer will buy the asset at any price if $s = NF$. Then $\mu^F(\mu) = \frac{\mu}{\mu+(1-\mu)(1-\overline{\sigma})}$, and $\mu = b$ if $P = P^b$ and $\mu = g$ if $P = P^g$ [2]. We solve the game by backward induction. Following a history of separation $\left(P^{b*}, P^g\right)$, second-period prices are given by $\left(P_2^{b*}, P_2^{g**}\right)$[3], where $P_1^{b*} = P_2^{b*} = P^{b*}$ is the maximizer of $\pi\left(b, P, \mu^F(b).\right)$ and P_2^{g**} is the maximizer of $\pi\left(g, P, \mu^F(g).\right)$. We denote by $\Pi\left(\theta, P, \mu^F(\mu)\right)$ the profits over both periods of a type-θ seller who sets the price P in the first period, inducing beliefs $\mu^F(\mu)$, conditional on a favorable inspection outcome.Thus equilibrium profits are given by

$$\Pi\left(b, P^{b*}, \mu^F(b)\right) = 2P^{b*}D\left(b, P^{b*}, \mu^F(b)\right)$$

$$\Pi\left(g, P^g, \mu^F(g)\right) = P^g D\left(g, P^g, \mu^F(g)\right) + P_2^{g**}D\left(g, P_2^{g**}, \mu^F(g)\right).$$

Proposition 9. In the dynamic game, with costless inspections, there is no separating equilibrium.

Proof. A separating equilibrium in $t = 1$ is a pair $\left(P^{b*}, P^g\right)$ such that two conditions simultaneously hold:

C1. $\Pi\left(b, P^{b*}, \mu^F(b)\right) \geq \Pi\left(b, P_1^g, \mu^F(g)\right)$, and

C2. $\Pi\left(g, P^g, \mu^F(g)\right) \geq \Pi\left(g, P^{g*}, \mu^F(b)\right)$.

[2] Note that $\mu_1^F = \mu_2^F = \mu^F(\mu)$.

[3] Second-period equlibrium prices are calculated by maximizing profits, since sellers' private information was fully revealed in the first period, where a separating equilibrium was played.

Separation can occur if the bad seller chooses its maximizing price rather than mimicking the good one, even if this implies optimistic beliefs (C1), and the good seller chooses not to monopolize the market by charging his maximizing price P^{g*} in the first period, being perceived as a bad seller (C2). The proof follows the same reasoning of Lemma 3 proof. Consider the equilibrium price $P^g = \overline{P} > P^{b*}$ such that $\Pi\left(b, P^{b*}, \mu^F(b)\right) = \Pi\left(b, \overline{P}, \mu^F(g)\right)$ (1). Condition (1) is equivalent to

$$2\left[P^{b*}\overline{D}\left(P^{b*}, \mu^F(b)\right)(b + (1-b)(1-\overline{\sigma}))\right] =$$
$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(b + (1-b)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(b + (1-b)(1-\sigma))$$

At this price the good seller should not have any incentive to deviate to his "monopoly" price, i.e. $\Pi\left(g, P_1^g, \mu^F(g)\right) > \Pi\left(g, P_1^{g*}, \mu^F(b)\right)$ (2):

$$2\left[P^{g*}\overline{D}\left(P^{g*}, \mu^F(b)\right)(g + (1-g)(1-\overline{\sigma}))\right] \leq$$
$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})).$$

By equality (1) this is equivalent to

$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) <$$
$$\overline{PD}\left(\overline{P}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu^F(g)\right)(g + (1-g)(1-\overline{\sigma})),$$ which cannot be true since $P^{b*} = P^{g*}$.

Thus there is no separating equilibrium satisfying both (1) and (2) at the same time. ■

Costly inspections and two-period game

We now consider the case in which the selling season is composed of two periods and there is an object on sale each period. When information acquisition is costly, the dynamic separating equilibrium shows higher separating prices than the static one.

Proposition 10. There is always a separating equilibrium $\left(P^{b*}, P_1^g\right)$ with $P_1^g \geq P^{g*}$ if $\frac{\sigma_P}{\sigma} > -\frac{\overline{D}_P}{\overline{D}}$ and $-\frac{\sigma_\mu}{\sigma} < \frac{\overline{D}_\mu}{\overline{D}}$, where $\frac{\sigma_P}{\sigma}\left(\frac{\sigma_\mu}{\sigma}\right)$ represents the elasticity of the information precision to the price (beliefs) and $\frac{\overline{D}_P}{\overline{D}}\left(\frac{\overline{D}_\mu}{\overline{D}}\right)$ the elasticity of demand to price (beliefs). Moreover the dynamic separating equilibrium shows higher separating prices than the static one, $P_1^g > P^g$.

Proof. The proof follows the same steps as for the static case (see Proof of Proposition 5). We now show that the dynamic separating equilibrium shows higher separating prices than the static one $P_1^g > P^g$. We defined $\overline{P}$ as the price at which the bad seller was indifferent between following the equilibrium strategy and mimicking the good one in the static game, $\pi\left(b, P^{b*}, \mu = b\right) = \pi\left(b, \overline{P}, \mu = g\right)$ (1). Now define $\widetilde{P}$ as its equivalent for the two-period game, the price such that $\Pi\left(b, P^{b*}, \mu = b\right) = \Pi\left(b, \widetilde{P}, \mu = g\right)$ (2). Expressions (1) and (2) can be written as

$$P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right) = \overline{P}\overline{D}\left(\overline{P}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(\overline{P}, \mu = g\right)\right)\right)$$

(1)

$$2\left[P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right)\right] =$$
$$\widetilde{P}\overline{D}\left(\widetilde{P}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(\widetilde{P}, \mu = g\right)\right)\right) + P_2^{g**}\overline{D}\left(P_2^{g**}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(P_2^{g**}, \mu = g\right)\right)\right)$$

(2)

Suppose that $\widetilde{P} = \overline{P}$. If this is the case, then condition (2) reduces to

$$P^{b*}\overline{D}\left(P^{b*}, \mu = b\right)\left(b + (1-b)\left(1 - \sigma\left(P^{b*}, \mu = b\right)\right)\right) =$$
$$P_2^{g**}\overline{D}\left(P_2^{g**}, \mu = g\right)\left(b + (1-b)\left(1 - \sigma\left(P_2^{g**}, \mu = g\right)\right)\right),$$

which cannot be true since $\pi\left(b, P_2^{g**}, \mu = g\right) > \pi\left(b, P^{b*}, \mu = b\right)$. Then, to mantain the equality in condition (2) it must be that $\widetilde{P} > \overline{P}$.■