

# THE CROSS-SECTIONAL VARIATION OF VOLATILITY RISK PREMIA

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## Abstract

This paper analyzes the determinants of the cross-sectional variation of the average volatility risk premia for a set of 20 portfolios sorted by volatility risk premium beta. The market volatility risk premium and, especially, the default premium are shown to be key determinants risk factors in the cross-sectional variation of average volatility risk premium payoffs. The cross-sectional variations of risk premia reflects the different uses of volatility swaps in hedging default and the financial stress risks of the underlying components of our sample portfolios.

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## 1. Introduction

Since the seminal paper of Bakshi and Kapadia (2003a), the market variance risk premium has been reported to be negative, on average, during alternative sample periods.<sup>1</sup> Since the payoff of a variance swap contract is the difference between the realized variance and the variance swap rate, negative returns to long positions on variance swap contracts for all time horizons mean that investors are willing to accept negative returns for purchasing realized variance.<sup>2</sup> Equivalently, investors who are sellers of variance and are providing insurance to the market, require substantial positive returns. This may be rational, since the correlation between volatility shocks and market returns is known to be strongly negative and investors want protection against stock market crashes. Along these lines, Bakshi and Madan (2006), and Chabi-Yo (2012) theoretically show that the skewness and kurtosis of the underlying market index are key determinants of the market variance risk premium. Indeed, Bakshi and Madan (2006), Bollerslev, Gibson, and Zhou (2011), Bekaert and Hoerova (2013), and Bekaert, Hoerova and Lo Duca (2013) argue that the market variance risk premium is an indicator of aggregate risk aversion.<sup>3</sup> A related interpretation is due to Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011), who interpret the market variance risk premium as a proxy of macroeconomic risk (consumption uncertainty). They show that time-varying economic uncertainty and a preference for the early resolution of uncertainty are required to generate a negative market variance risk premium. Zhou (2010) shows that the market variance risk premium significantly predicts short-run equity returns, bond returns, and credit spreads. Consequently, the

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<sup>1</sup> For empirical evidence of the negative variance risk premium on the market index, see Carr and Wu (2009) and the papers cited in their work.

<sup>2</sup> A variance swap is an over-the-counter derivative contract in which two parties agree to buy or sell the realized variance of an index or single stock on a future date.

<sup>3</sup> More specifically, Bekaert, Hoerova, and Lo Duca (2013) show the interactions between monetary policy and the market variance risk premium, suggesting that monetary policy may impact aggregate risk aversion.

author argues that risk premia in major markets comove in the short- run and that such comovement seems to be related to the market variance risk premia. Campbell, Giglio, Polk, and Turley (2014), using an intertemporal capital asset pricing model (CAPM) framework, argue that covariation with aggregate volatility news has a negative premium. Finally, Nieto, Novales, and Rubio (2014) show that the uncertainty that determines the variance risk premium –the investors’ fear of deviating from normality in returns– is also strongly related to a variety of macroeconomic and financial risks associated with default, employment growth, consumption growth, and stock market and market illiquidity risks. At this point, it is fair to argue that we understand the behavior of the market variance risk premium and its implications for financial economics.

However, it is surprising how little we know about the variance risk premium at the individual level. Bakshi and Kapadia (2003b) show that the variance risk premium is also negative in individual equity options. However, Driessen, Maenhout, and Vilkov (2009) show that the variance risk premium for stock indices is systematically larger, that is, more negative, than for individual securities. They argue that the variance risk premium can, in fact, be interpreted as the price of time-varying correlation risk. They show that the market variance risk is negative only to the extent that the price of the correlation risk is negative. In a related paper, Buraschi, Trojani, and Vedolin (2014) argue that the wedge between index and volatility risk premia is explained by investor disagreement. Hence, the greater the differences in beliefs among investors, the larger the market volatility risk relative to the volatility risk premium of individual options. Even these papers are particularly concerned with the behavior of the market variance risk premium, despite employing data at the individual level.

We argue that an analysis and the understanding of the time-series and cross-sectional behavior of the variance risk premium at the individual level is lacking in the previous literature. This paper partially covers this gap. More specifically, we analyze the cross-sectional variation of the volatility risk premium (*sVRP*) at the portfolio level. We employ daily data from OptionMetrics for the Standard & Poor's (S&P) 100 Index options and for individual options on 181 stocks included at some point in the S&P 100 Index during the sample period from January 1996 to February 2011. We employ options with one month to expiration. We calculate *sVRP* for each stock at the 30-day horizon as the difference between the corresponding realized volatility and the model-free implied volatility described by Jiang and Tian (2005). Similarly, we estimate the market volatility risk premium using the S&P 100 Index as the underlying index. For each month, using an individual *sVRP* with at least 15 daily observations, we construct 20 equally weighted portfolios ranking the individual *sVRP* values according to their betas with respect to the market *sVRP*. These volatility risk premium betas are estimated over the previous month with daily data. Although we briefly describe the time-varying behavior of volatility risk premia for our 20 *sVRP* beta-sorted portfolios and their betas with respect to alternative aggregate sources of risk, the main objective of the paper is to analyze the determinants of the cross-sectional variation of average volatility risk premia across our sample of 20 portfolios.

We find that the betas of the *sVRP* beta-sorted portfolios estimated with respect to the market *sVRP*, obtained from the S&P 100 Index options, range from -0.95 to 3.89, where the portfolio with the most negative beta has the highest average *sVRP* and the portfolio with the most positive beta presents the most negative average *sVRP*. Therefore, we find both negative and positive average *sVRP* values ranging from 0.103

to -0.034 on an annual basis, while the average market  $sVRP$  is negative, as in previous literature.

Regarding the cross-sectional variation of the volatility risk premia, we find that, independently of the preferences imposed, consumption risk does not seem to explain the cross-sectional behavior of  $sVRP$ . Factor asset pricing models seem to be more useful in explaining  $sVRP$  at the cross section. The key factors explaining average  $sVRP$  across our 20 portfolios are the market volatility risk premium and, especially, the default premium. The risk premia associated with the default premium betas are positive and statistically significant even if we explicitly recognize the potential misspecification of the models. Moreover, we cannot reject the overall specification of the two-factor model and the cross-sectional  $R^2$  is equal to 0.514, with an asymptotic standard error of 0.211. Finally, our findings are related to credit risk and financial market stress conditions. More precisely, the cross-sectional variations of risk premia reflects the different uses of volatility swaps to hedge default and the financial stress risks of the underlying components of our sample portfolios.

This paper is organized as follows. Section 2 briefly describes variance swaps and volatility swap contracts and presents the alternative asset pricing models that we employ in the study of the cross-sectional variation of average  $sVRP$ . Section 3 contains a description of the data. Section 4 discusses the model-free implied volatility and the estimation of  $sVRP$  at the portfolio level. Section 5 presents the basic characteristics of the 20  $sVRP$  beta-sorted portfolios and some empirical results using unconditional  $sVRP$  beta estimates. Section 6 reports the main empirical findings of the paper and discusses the econometric strategy. Section 7 relates our evidence to financial stress conditions. Section 8 concludes the paper.

## 2. Theoretical Framework

In a variance swap, the buyer of this forward contract receives at expiration a payoff equals to the difference between the annualized variance of stock returns and the fixed swap rate. The swap rate is chosen such that the contract has zero present value, which implies that the variance swap rate represents the risk-neutral expected value of the realized return variance:

$$E_t^Q(RV_{t,t+\tau}^a) = SW_{t,t+\tau}^a \quad (1)$$

where  $E_t^Q(\cdot)$  is the time  $t$  conditional expectation operator under some risk-neutral measure  $Q$ ,  $RV_{t,t+\tau}^a$  is the realized variance of asset (or portfolio)  $a$  between  $t$  and  $t + \tau$ , and  $SW_{t,t+\tau}^a$  is the delivery price for the variance or the variance swap rate on the underlying asset  $a$ . The variance risk premium of asset  $a$  is defined as

$$VRP_{t,t+\tau}^a = E_t^P(RV_{t,t+\tau}^a) - E_t^Q(RV_{t,t+\tau}^a) \quad (2)$$

On the other hand, at expiration, a volatility swap pays the holder the difference between the annualized volatility and the volatility swap rate,

$$N_{vol} (sRV_{t,t+\tau}^a - sSW_{t,t+\tau}^a) \quad (3)$$

where  $sRV_{t,t+\tau}^a$  is the realized volatility of asset  $a$  between  $t$  and  $t + \tau$ ,  $sSW_{t,t+\tau}^a$  is the fixed volatility swap rate, and  $N_{vol}$  denotes the volatility notional. This paper analyzes the determinants of the cross-sectional variation of volatility risk premia. We therefore define the volatility risk premium of asset  $a$  as follows,

$$sVRP_{t,t+\tau}^a = E_t^P(sRV_{t,t+\tau}^a) - E_t^Q(sRV_{t,t+\tau}^a) \quad (4)$$

Using the fundamental asset pricing equation, we know that the risk premium of any asset  $a$  with rate of return  $R_t^a$  is given by

$$RP_{t,t+\tau}^a = -\frac{Cov_t^P(M_{t,t+\tau}, R_{t,t+\tau}^a)}{E_t^P(M_{t,t+\tau})} \quad (5)$$

where  $M_{t,t+\tau}$  is the stochastic discount factor (SDF). Therefore, given the definition of the volatility risk premium, the following expression holds:

$$E_t^Q(sRV_{t+\tau}^a) = E_t^P(sRV_{t+\tau}^a) + \frac{Cov_t^P(M_{t,t+\tau}, sRV_{t,t+\tau}^a)}{E_t^P(M_{t,t+\tau})} \quad (6)$$

Thus, using the payoff of a volatility swap, the fundamental pricing framework implies that

$$E_t^P \left[ M_{t,t+\tau} (sRV_{t,t+\tau}^a - sSW_{t,t+\tau}^a) \right] = E_t^P \left[ M_{t,t+\tau} (sVRP_{t,t+\tau}^a) \right] = 0 \quad (7)$$

In this paper, the SDF,  $M_{t,t+\tau}$ , is allowed to be based on either power, recursive, and habit preferences or on alternative linear SDF specifications based on state variables potentially capable of explaining the cross-sectional variation of volatility swaps. In particular, we test the following models:

a) Model 1, C1, power utility with aggregate consumption:

$$M_{t,t+\tau} = \rho \frac{U'(C_{t+\tau})}{U'(C_t)} = \rho \left( \frac{C_{t+\tau}}{C_t} \right)^{-\gamma} \quad (8a)$$

where  $C_t$  is the aggregate consumption of non-durable goods and services,  $\gamma > 0$  represents the degree of risk aversion, and  $\rho$  is the subjective discount factor.

b) Model 1, C2, power utility with stockholder consumption, denoted  $C_t^{SHC}$ :

$$M_{t,t+\tau} = \rho \left( \frac{C_{t+\tau}^{SHC}}{C_t^{SHC}} \right)^{-\gamma} \quad (8b)$$

c) Model 2, C1, recursive utility with aggregate consumption:

$$U_t = \left[ (1-\rho)C_t^{1-\kappa} + \rho \left( E_t \left( U_{t+\tau}^{1-\gamma} \right) \right)^{\frac{1-\kappa}{1-\gamma}} \right]^{\frac{1}{1-\kappa}} \quad (9)$$

where the non-observable continuation value is approximated, as for Epstein and Zin (1991), by the return on the market portfolio or market wealth so that the corresponding SDF becomes

$$M_{t,t+\tau} = \left[ \rho \left( \frac{C_{t+\tau}}{C_t} \right)^{-\kappa} \right]^{\eta} \left[ \frac{1}{R_{mt+\tau}} \right]^{\frac{\gamma-\kappa}{1-\kappa}} \quad (10a)$$

where  $\eta = \frac{1-\gamma}{1-\kappa}$  and  $\kappa$  is the inverse of the elasticity of intertemporal substitution.

d) Model 2, C2, recursive utility with stockholder consumption:

$$M_{t,t+\tau} = \left[ \rho \left( \frac{C_{t+\tau}^{SHC}}{C_t^{SHC}} \right)^{-\kappa} \right]^{\eta} \left[ \frac{1}{R_{mt+\tau}} \right]^{\frac{\gamma-\kappa}{1-\kappa}} \quad (10b)$$

e) Model 3, C1, external habit preferences, as for Campbell and Cochrane (1999):

$$U_t = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \quad (11)$$

where  $X_t$  is the level of habit and the SDF is given by

$$M_{t,t+\tau} = \rho \left( \frac{S_{t+\tau} C_{t+\tau}}{S_t C_t} \right)^{-\gamma} \quad (12)$$

where  $\gamma$  is a parameter of utility curvature,  $S_t = C_t - X_t/C_t$  is the surplus consumption ratio, and the counter-cyclical time-varying risk aversion is given by  $\gamma/S_t$ . The aggregate consumption follows a random walk and the surplus consumption process is

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - g) \quad (13)$$



where  $g$  is the mean rate of consumption growth,  $\phi$  is the persistence of the habit shock, and the response or sensitivity coefficient  $\lambda(s_t)$  is given by

$$\lambda(s_t) = \left(1/\sigma_c \sqrt{\gamma/1-\phi}\right) \sqrt{1-2(s_t - \bar{s})} - 1 \quad (14)$$

where  $\sigma_c$  is the volatility of the consumption growth rate and lower capital letters denote variables in logarithms.

f) Model 3, C2, external habit with stockholder consumption:

$$M_{t,t+\tau} = \rho \left( \frac{S_{t+\tau}^{SHC} C_{t+\tau}^{SHC}}{S_t^{SHC} C_t^{SHC}} \right)^{-\gamma} \quad (15)$$

g) Model 4, C1, recursive preferences with the market volatility risk premium as the continuation value:

$$M_{t,t+\tau} = \left[ \rho \left( \frac{C_{t+\tau}}{C_t} \right)^{-\kappa} \right]^\theta \left[ \frac{1}{sVRP_{t+\tau}^m} \right]^{\frac{\gamma-\kappa}{1-\kappa}} \quad (16)$$

where  $sVRP_{t+\tau}^m$  is the market volatility risk premium.

h) Model 4, C2, recursive preferences with the market volatility risk premium, and stockholder consumption:

$$M_{t,t+\tau} = \left[ \rho \left( \frac{C_{t+\tau}^{SHC}}{C_t^{SHC}} \right)^{-\kappa} \right]^\theta \left[ \frac{1}{sVRP_{t+\tau}^m} \right]^{\frac{\gamma-\kappa}{1-\kappa}} \quad (17)$$

i) Model 5: linear SDF for both the market return and the squared of aggregate wealth:

$$M_{t,t+\tau} = a + bR_{mt+\tau} + cR_{mt+\tau}^2 \quad (18)$$

As previously discussed, recent empirical work has consistently shown that risk-neutral volatility is higher, on average, than physical return volatility. Little work has been done on theoretically characterizing the distance between both types of volatility, with Bakshi and Madan (2006) and Chabi-Yo (2012) being two exceptions. In both cases, the market

variance risk premium is derived as a function of the standard deviation, skewness, and kurtosis of equity returns. Therefore, the magnitude and behaviour over time of the market variance risk premium may also be empirically related to higher -order moments of the equity return distribution. This suggests that a potentially relevant model to explain the cross-sectional variation of volatility risk premia should explicitly recognize higher-order moments of the underlying market portfolio return. In particular, Bakshi and Madan (2006) show that, when the SDF is a linear function on both the market return and the squared of market return, as in expression (18), then the variance risk premium is a function of both the skewness and kurtosis of the market and  $\partial M/\partial R_m < 0$  and  $\partial^2 M/\partial R_m^2 > 0$ .

j) Model 6: CAPM with the market volatility risk premium:

$$M_{t,t+\tau} = a + bsVRP_{t+\tau}^m \quad (19)$$

This may be justified by noting that Bali and Zhou (2012) show that the cross- section of equity returns portfolios is explained by the market, and also by economic uncertainty proxied by the market variance risk premium.

k) Model 7: multi-factor SDF with the market volatility risk premium and the default premium as the difference between the Moody's yield on Baa corporate bonds and the 10-year government bond yield, denoted  $DEF_{t+\tau}$  :

$$M_{t,t+\tau} = a + bsVRP_{t+\tau}^m + cDEF_{t+\tau} \quad (20)$$

The economic rationale of this model comes from the findings of Zhou (2010) and Wang, Zhou, and Zhou (2013), who show that the firm-level variance risk premium has significant explanatory power for credit default swap spreads over and above the market variance risk premium and the VIX. The predictive ability increases as the credit quality of the credit default swap underlying companies deteriorates.

All these SDF specifications will be tested using a generalized method of moments (GMM) framework with the same weighting matrix across all test portfolios to compare the performance of the models by the Hansen–Jagannathan (1997, henceforth HJ) distance. Additionally, we employ the two-pass cross-sectional regression approach of Fama and MacBeth (1973). In this case, we use the linear versions of all previous discussed models and also include the simple CAPM with the market portfolio return and extended models using the market portfolio return, the market volatility risk premium, the Fama–French HML factor, and the default premium as additional pricing factors.

### **3. Data**

We employ daily data from OptionMetrics for the S&P 100 Index options and for individual options on all stocks included in the S&P 100 Index at some point during the sample period from January 1996 to February 2011. This yields a total of 181 stocks used in our estimations. From the OptionMetrics database, we obtain all put and call options on the individual stocks and on the index with time to maturity between six days and 90 days. Given that the options are American style, it is convenient to work with short-term maturity options, for which the early exercise premium tends to be negligible.<sup>4</sup> We select the best bid and ask closing quotes to calculate the mid-quotes as the average of bid and ask prices, rather than actual transaction prices, to avoid the well known bid–ask bounce problem described by Bakshi, Cao, and Chen (1997). In selecting our final option sample, we apply the usual filters. We discard options with zero open interest, zero bid prices, missing delta or implied volatility, and negative implied volatility. We also ignore options with extreme moneyness, that is, puts with a

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<sup>4</sup> See the evidence reported by Driessen, Maenhout, and Vilkov (2009) who employ a similar database.

Black–Scholes delta above  $-0.05$  and calls with a delta below  $0.05$ . Finally, regarding the exercise level, we employ out-of-the-money options using puts with a delta above  $-0.5$  and calls with a delta below  $0.5$ .

It seems reasonable to expect that aggregate macroeconomic variables and market-wide portfolios extensively used by researchers when explaining the time series and cross-sectional behavior of excess equity returns should also be relevant factors in explaining variance risk premia across assets. This is the main criterion we follow when collecting our data. As our option data, the market return for the S&P 100 Index and individual stock returns and dividends are also obtained from OptionMetrics, while portfolio return data are from Kenneth French’s website. In particular, we collect monthly data on the value-weighted stock market portfolio return, the risk-free rate, the SMB and HML Fama–French risk factors, and the momentum factor denoted MOM.

Additionally, yields for 10-year government bonds, 1-month T-bills, and Moody’s Baa corporate bonds are obtained from the Federal Reserve Statistical Release. The default premium, denoted DEF, is the difference between Moody’s yield on Baa corporate bonds and the 10-year government bond yield.

We obtain nominal consumption expenditures on nondurable goods and services from Table 2.8.5 of the National Income and Product Accounts (NIPA), available at the Bureau of Economic Analysis. Population data are from NIPA’s Table 2.6 and the price deflator is computed using prices from NIPA’s Table 2.8.4, with the year 2000 as its basis. All this information is used to construct monthly rates of growth of seasonally adjusted real per capita consumption expenditures on nondurable goods and services from January 1959 to September 2012. We also use aggregate per capita stockholder consumption growth rates. Exploiting micro-level household consumption data, Malloy, Moskowitz, and Vissing-Jorgensen (2011) show that long-run stockholder consumption

risk explains the cross-sectional variation in average stock returns better than the aggregate consumption risk obtained from nondurable goods and services. In addition, they report plausible risk aversion estimates. They employ data from the Consumer Expenditure Survey (CEX) for the period March 1982 to November 2004 to extract consumption growth rates for stockholders, the wealthiest third of stockholders, and non-stockholders. To extend their available time period for these series, the authors construct factor-mimicking portfolios by projecting the stockholder consumption growth rate series from March 1982 to November 2004 onto a set of instruments and use the estimated coefficients to obtain a longer time series of instrumented stockholder consumption growth. In this paper, we employ the reported estimated coefficients of Malloy, Moskowitz, and Vissing-Jorgensen (2011) to obtain a factor-mimicking portfolio with the same set of instruments for stockholder consumption from January 1960 to September 2012.

#### 4. Model-Free Implied Volatility and Estimation of the Volatility Risk Premia

Britten-Jones and Neuberger (2002) first derived the model-free implied volatility under diffusion assumptions. They obtain the risk-neutral expected integrated variance over the life of the option contract when prices are continuous and volatility is stochastic. Jiang and Tian (2005) extend their paper to show that their method is also valid in a jump-diffusion framework and, therefore, their methodology is considered to be a model-free procedure.

We calculate the model-free implied variance denoted  $MFIV_{t,t+\tau}^a$  by the following integral over a continuum of strikes:

$$MFIV_{t,t+\tau}^a = 2 \int_0^{\infty} \frac{C_{t,t+\tau}^a(K)/B(t,t+\tau) - \max(S_t^a/B(t,t+\tau) - K, 0)}{K^2} dK \quad (21)$$

where  $C_{t,t+\tau}^a(K)$  is the spot price at time  $t$  of a  $\tau$ -maturity call option on either an asset or index  $a$  with strike  $K$ ,  $B(t,t+\tau)$  is the time  $t$  price of a zero-coupon bond that pays \$1 at time  $t + \tau$ , and  $S_t^a$  is the spot price of asset  $a$  at time  $t$  minus the present value of all expected future dividends to be paid before the option maturity. Expression (21) can be accurately approximated by the following sum over a finite number of strikes:

$$MFIV_{t,t+\tau}^a \cong \sum_{j=1}^m \left[ g_{t,t+\tau}^a(K_j) + g_{t,t+\tau}^a(K_{j-1}) \right] \Delta K \quad (22)$$

where

$$\Delta K = \frac{(K_{max} - K_{min})}{m}, \quad K_j = K_{min} + j\Delta K \quad \text{for } j=0,1,\dots,m$$

and

$$g_{t,t+\tau}^a(K_j) = \frac{C_{t,t+\tau}^a(K_j)/B(t,t+\tau) - \max(S_t^a/B(t,t+\tau) - K_j, 0)}{K_j^2}$$

For each time -to- maturity from six days to 60 days, we calculate the model-free implied variance each day for each underlying asset that has at least three available options outstanding, using all the available options at time  $t$ .<sup>5</sup> For the risk-free rate, we use the T-bill rate of appropriate maturity (interpolated when necessary) from OptionMetrics, namely, the zero-coupon curve. For the dividend rate for the index we employ the daily data on the index dividend yield from OptionMetrics. To infer the continuously compounded dividend rate for each individual asset, we combine the forward price with the spot rate used for the forward price calculations. We obtain the mean continuously compounded dividend rate by averaging the implied OptionMetrics

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<sup>5</sup> The window from six days to 60 days corresponds to the maximum range of time to maturity we allow in the necessary interpolation to have enough options every day in the sample with 30 days to maturity. See the discussion below.

dividends. Finally, we annualize the model-free implied variance using 252 trading days in a calendar day.

The specific implementation follows the approach of Jiang and Tian (2005). It is well known that options are traded only over a limited number of strikes. In principle, expression (22) requires the prices of options with strikes  $K_j$  for  $j = 0, 1, \dots, m$ . However, the corresponding option prices are not observable because these options are not listed. We apply the curve-fitting method to Black–Scholes implied volatilities instead of option prices. The prices of listed calls (and puts with different strikes) are first transformed into implied volatilities using the Black–Scholes model and a smooth function is fitted to the implied volatilities using cubic splines.<sup>6</sup> Then, we extract implied volatilities at strikes  $K_j$  from the fitted function. Finally, we employ equation (22) to calculate the model-free implied variance using the extracted option prices.

It is sometimes the case that the range of available strikes is not sufficiently large. For option prices outside the range between the maximum and minimum available strikes, we also follow Jiang and Tian (2005) and use the endpoint implied volatility to extrapolate their option prices. This implies that the volatility function is assumed to be constant beyond the maximum and minimum strikes.<sup>7</sup> Finally, discretization errors are unlikely to have any effect on the model-free implied variance if a sufficiently large  $m$ , beyond 20, is chosen. In our case, we employ an  $m$  that equals 100.

At each time  $t$ , we focus on a 30-day horizon maturity, interpolated when necessary using the nearest maturities  $\tau_1$  and  $\tau_2$  following the procedure of Carr and Wu (2009). The interpolation is linear in total variance:

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<sup>6</sup> As pointed out by Jiang and Tian (2005), the curve-fitting procedure does not assume that the Black–Scholes model holds. It is a tool to provide a one-to-one mapping between prices and implied volatilities.

<sup>7</sup> Jiang and Tian (2005) discuss this approximation error and the (different) truncation error that arise when we ignore the tails of the distribution across strikes. In our case and to avoid the truncation error, we use 3.5 standard deviations from the spot underlying price as truncation points.

$$MFIV_{t,t+\tau}^a = \frac{1}{\tau} \left[ \frac{MFIV_{t,t+\tau_1}^a \tau_1 (\tau_2 - \tau) + MFIV_{t,t+\tau_2}^a \tau_2 (\tau - \tau_1)}{(\tau_2 - \tau_1)} \right] \quad (23)$$

We use the square root of the model-free implied variance to approximate the model-free annualized implied volatility as:

$$sMFIV_{t,t+\tau}^a = \sqrt{MFIV_{t,t+\tau}^a} \quad (24)$$

For each day in the sample period, we also calculate the realized variance over the same period as that for which implied variance is obtained for that day, that is, for 30 days, requiring that no more than 14 returns be missing from the sample:

$$RV_{t,t+\tau}^a = \frac{1}{\tau} \sum_{s=1}^{\tau} R_{t+s}^2 \quad (25)$$

where  $R$  denotes the rate of return adjusted by dividends and splits. As before, we annualized the realized variance and take the square root to obtain the realized volatility:

$$sRV_{t,t+\tau}^a = \sqrt{RV_{t,t+\tau}^a} \quad (26)$$

Finally, for each asset and the index, we calculate the volatility risk premium,  $sVRP$ , at the 30-day horizon as the difference between the corresponding realized and model-free implied volatility:

$$sVRP_{t,t+\tau}^a = sRV_{t,t+\tau}^a - sMFIV_{t,t+\tau}^a \quad (27)$$

We next construct 20  $sVRP$  beta-sorted portfolios using the following procedure. We estimate rolling  $sVRP$  betas for each month using daily data over the previous month on the individual  $sVRP$  and the market  $sVRP$ . Each month, we rank all  $sVRP$  betas and construct 20 equally weighted  $sVRP$  beta-sorted portfolios. Portfolio 1 contains the most negative  $sVRP$  betas, while Portfolio 20 includes the most positive  $sVRP$  betas. The components of all portfolios are updated every month during the sample period. All



portfolios have approximately the same number of securities, with an average of 5.3 securities per portfolio, and the asset must have at least 15 daily observations to be included in the portfolios.

Figure 1 displays the behavior of portfolios 1, 10, and 20 sorted by *sVRP* beta, as well as the market *sVRP*. Note that we display the *sVRP* of the market using options written on the S&P 100 Index, so that the series contained in Figure 1 is not the cross-sectional average of the individual *sVRP*. For the portfolios P10B and P20B and the market, the positive peaks coincide with periods of high realized volatility. Portfolio P1B tends to have a positive *sVRP* even during normal economic times, while portfolio P20B presents a negative *sVRP* during normal and expansion months and a positive *sVRP* during bad economic times. As expected, given that the *sVRP* beta of portfolio P20B is as high as 3.89, its behavior closely follows the market *sVRP*, but with more extreme peaks. In any case, this figure suggests that the ranking procedure generates sufficiently different cross-sectional behaviour to justify the analysis of the cross-sectional empirical results under this sorting characteristic.<sup>8</sup>

## 5. Volatility Risk Premium Characteristics at the Portfolio Level

Table 1 reports the basic characteristics of our 20 *sVRP* beta-sorted portfolios. The average *sVRP* values are 10.3% and -3.4% for portfolios P1B and P20B, respectively. All of these figures are given in annualized terms. As expected, given the well-known evidence provided, among others, by Carr and Wu (2009), the market *sVRP* is, on average, negative and equal to -1.4%. The average annualized *sVRP* obtained directly from daily data present a very similar pattern, with the range going from 10.1% to

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<sup>8</sup> We also construct an alternative set of 20 portfolios based on the *sVRP* level. Using the *sVRP* on the last day of the previous month, we rank all *sVRP* values from the lowest (more negative) to the highest. Portfolio 1 contains the assets with the lowest *sVRP*, while portfolio 20 includes securities with the highest *sVRP*. Our main empirical results and conclusions will be checked employing this alternative ranking to analyze the robustness of our results.

-4.5%. The magnitude of the  $sVRP$  cross-sectional differences is large and seems to justify the study of their determinants. These averages indicate that investors may have very different volatility investment vehicles depending on whether they go long or short on volatility. We tend to identify the purchase of volatility as a hedging instrument against potentially large stock market declines. The evidence reported in Table 1 suggests that, on average, going long on volatility can also lead to substantial gains, depending on the portfolio for which investors buy volatility.<sup>9</sup> The standard deviations of the  $sVRP$  values of these portfolios suggest that portfolios with a higher average  $sVRP$  and, especially, those with a more negative average  $sVRP$  are the most volatile portfolios in terms of  $sVRP$  payoffs. As pointed out before, Figure 1 also reflects the highly volatile behavior of the  $sVRP$  of P20B, followed by the relatively smoother behavior of P1B.

The fifth column of Table 1 contains the  $sVRP$  betas of each of the portfolios relative to the  $sVRP$  of the market index. Using monthly data, we estimate a market model type of ordinary least squares (OLS) regression of the following form:

$$sVRP_{t,t+\tau}^p = a + \beta sVRP_{t,t+\tau}^m + \varepsilon_{t,t+\tau} , \quad (28)$$

where  $sVRP_{t,t+\tau}^p$  is the volatility risk premium of each of the 20 portfolios, and

$sVRP_{t,t+\tau}^m$  is the volatility risk premium of the market index from January 1996 to February 2011. The  $sVRP$  betas reflect the construction criterion, with unconditional  $sVRP$  betas of -0.95 for P1B and 3.89 for P20B. As in the case of average volatility risk premia, the cross-sectional differences in  $sVRP$  betas are large.

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<sup>9</sup> As discussed by Carr and Lee (2007, 2009), due to the concavity's price impact associated with Jensen's inequality, the difference between the value of a variance swap and the value of a volatility swap depends on the volatility of volatility of the underlying asset. If we recognize this potential bias and adjust our estimated volatility risk premia accordingly, the dispersion between the volatility risk premia across portfolios remains. See Burashi, Trojani, and Vedolin (2014) for a similar approximation.

Given that, for each month during the sample period, we can identify the underlying components of the 20 portfolios, we calculate the portfolio returns of the 20 *sVRP* beta-sorted portfolios. In Table 1, we also display the market betas of the 20 portfolios with respect to the US market portfolio index and the S&P 100 Index. As with the standard deviation, the cross-sectional behavior of market betas presents a U-shaped pattern, with market betas being especially high for portfolios with a more negative average *sVRP*. Portfolio P20B has the highest return beta, with a value as high as 1.52 when measured relative to the S&P 100 Index return.

Finally, the last column of Table 1 contains the average relative bid–ask spread of the options associated with the components of the 20 portfolios. The options traded on the components of portfolios with positive and high average *sVRP* values may be extremely illiquid. If this is the case, the replicating strategy employed to obtain synthetic variance swaps associated with illiquid options may be more costly than in other cases. However, the average bid–ask spreads reflects precisely the opposite. The portfolio P1B contains, on average, the most liquid options, while P20B presents the highest relative bid–ask spread across the 20 portfolios. Therefore, on average, market return betas and bid–ask spreads are higher for the two portfolios with the highest *sVRP* betas.

Table 2 contains the correlation coefficients between representative portfolios sorted by *sVRP* betas and the market *sVRP*. Panel A employs monthly data, while Panel B displays the results with daily data. As expected, given its highly negative *sVRP* beta, the correlation between portfolio P1B and the rest of the portfolios becomes increasingly negative. Not surprisingly, the correlation of these portfolios with the market *sVRP* displays an increasingly monotonic relation going from a negative

correlation of -0.366 for P1B to a positive correlation of 0.863 for P20B. A similar pattern is found when using daily data.

Table 3 reports the correlation between the market *sVRP* and several macroeconomic and financial indicators. The correlation between the excess market return and the market *sVRP* is negative and equals -0.273. This is well known and implies a negative correlation between market returns and realized market volatilities. Thus, going long on the market volatility swap provides a hedging investment vehicle for moments of extremely high market volatility. However, the compensation for this hedging strategy is, on average, negative. The results also show a negative correlation of the market *sVRP* with consumption growth, although the correlation is more negative for aggregate consumption than for stockholder consumption. The correlation with the HML and momentum factors is positive, while the correlation with the default premium is also positive and equals 0.075. As expected, the correlation between the default premium and either the excess market return or consumption growth is negative, being especially negative with respect to aggregate consumption growth.

Panels A and B of Table 4 contain the full-sample *sVRP* betas for five representative *sVRP* beta-sorted portfolios controlling for well-known aggregate risk factors. The robustness of the magnitudes of the *sVRP* betas, reported again in the first column of Table 4, is clear across all portfolios. Independently of the factors employed in the regressions, portfolio P1B has a negative beta, while P20B has a very high but positive volatility risk premium beta. In all cases, we employ heteroskedasticity-autocorrelation (HAC) robust standard errors. The relation between the *sVRP* betas and the average volatility risk premia of all portfolios is maintained across all aggregate factors. We may conclude that, for *sVRP* beta-sorted portfolios, the volatility risk premia are especially explained by the market *sVRP*, the excess market return, the

default premium, and consumption growth. However,  $sVRP$  betas do not seem to be significantly different from zero when stockholder consumption growth is used. Overall, we conclude that the unconditional betas of these state variables are, in most cases, statistically different from zero, even when we employ all three explanatory variables simultaneously.

## 6. Cross-Sectional Variation of Portfolio Volatility Risk Premia

### 6.1 GMM Estimation and Tests

We next test the competing specifications given by models 1 through 7 described in Section 2 using the GMM estimation procedure and our set of 20 portfolios as test assets. Given the theoretical framework of Section 2, we work with the volatility risk premia of the 20  $sVRP$  beta-sorted portfolios. We define an  $(N+1) \times 1$  vector containing the pricing errors generated by the model at time  $t$ . The first  $N$  conditions are the pricing errors of the model when explaining the volatility risk premia of  $N$  portfolios. The last condition forces the SDF to go to its mean value  $\mu$ . More precisely and using the fundamental pricing equation given by (7), the following vector defines the moment restrictions:

$$f_t(\theta, \alpha, \mu) = E \begin{bmatrix} sVRP_t - \alpha I_N + \frac{(M_t - \mu) sVRP_t}{\mu} \\ M_t - \mu \end{bmatrix} \quad (29)$$

where  $sVRP_t$  is the  $N \times 1$  vector of volatility risk premia of the  $N$  portfolios at time  $t$ ,  $I_N$  denotes an  $N \times 1$  vector of ones,  $M_t(\theta)$  is one out of the seven specifications of equations (8) to (20), and  $\theta$  is the vector of the preference parameters for each

particular specification.<sup>10</sup> The inclusion of the parameter  $\alpha$  enables the separate evaluation of the model's ability to explain the temporal pricing behavior of the competing specifications and the cross section of volatility risk premia. So, if  $\alpha$  is zero, we can conclude that the model presents a zero average pricing error over the sample period. We define a vector containing the sample averages corresponding to the elements of  $f$  as

$$g_T(\theta, \alpha, \mu) = \frac{\sum_{t=1}^T f_t(\theta, \alpha, \mu)}{T} \quad (30)$$

and the GMM minimizes the quadratic form,

$$g_T(\theta, \alpha, \mu)' W_T g_T(\theta, \alpha, \mu) \quad (31)$$

where  $W_T$  is a weighting  $(N+1) \times (N+1)$  matrix.

For the GMM estimation and to compare the performance of the models, we employ the pre-specified weighting matrix that contains the matrix proposed by HJ. It weights the moment conditions for the  $N$  testing portfolios using the (inverse) matrix of second moments of the volatility risk premia of our set of 20 portfolios. Moreover, as for Parker and Julliard (2005), the weight of the last moment condition is chosen large enough to ensure that significant changes in that weight have no effects on the parameter estimates. A weight of 1000 for the last moment condition ensures the stability of the estimator for the mean of the SDF with respect to different initial conditions. Hence, the pre-specified weighting matrix is

$$W_T = \begin{bmatrix} HJ & 0_N \\ 0_N & 1000 \end{bmatrix} \quad (32)$$

where

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<sup>10</sup> See Parker and Julliard (2005), and Yogo (2006) for examples of GMM estimation using the same estimation strategy. In the empirical estimation, we take the subjective discount rate as a fixed parameter that is equal to the inverse of the risk-free rate over the sample period.

$$HJ = \left[ \left( \frac{1}{T} \right) \sum_{t=1}^T sVRP_t sVRP_t' \right]^{-1} \quad (33)$$

and  $0_N$  is an  $N$ -dimensional vector of zeros. Given the unknown distribution of the performance test, we follow Jagannathan and Wang (1996), HJ and Parker and Julliard (2005) to infer the  $p$ -value of the test. The evaluation of the model performance is carried out by testing the following null hypothesis:

$$H_0 : T[\delta(\theta, \alpha, \mu)]^2 = 0 \quad (34)$$

where the HJ distance is defined as

$$\delta = \sqrt{g_T(\theta, \alpha, \mu)' W_T g_T(\theta, \alpha, \mu)} \quad (35)$$

It is well known that a limitation of the HJ distance in comparing asset pricing models is that it does not allow for statistical comparison among competing models. Chen and Ludvigson (2009) propose a procedure that can be used to compare any number of multiple competing models, some of them possibly non-linear. The benchmark model is the model with smallest squared HJ distances among competing models. The authors are able to compute the distribution of the differences between squared HJ distances via a block bootstrap, where the reference distance corresponds to that with the smallest HJ distance among all models. Kan and Robotti (2009, KR hereafter) also develop a methodology to test whether two competing models have the same HJ distance and they show that the asymptotic distribution of the test statistic depends on whether the models are correctly specified or not. In this paper, we apply the KR test of the comparison of the HJ distances of two alternative specifications under potentially misspecified models.<sup>11</sup>

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<sup>11</sup> We employ a version of their test for which the SDF does not have to be necessarily linear.

We briefly described their comparison test which amounts to obtaining the asymptotic distribution of  $\hat{\delta}_1^2 - \hat{\delta}_2^2$ . Let  $d_t = s_{1t} - s_{2t}$ , where

$$s_{1t} = 2\phi_1' sVRP_t M_{1t} - \left(\phi_1' sVRP_t\right)^2 - 2\phi_1' 1_N - \delta_1^2, \text{ where } \phi_1 = Wg_1$$

$$s_{2t} = 2\phi_2' sVRP_t M_{2t} - \left(\phi_2' sVRP_t\right)^2 - 2\phi_2' 1_N - \delta_2^2, \text{ where } \phi_2 = Wg_2$$

Under the null hypothesis of  $\hat{\delta}_1^2 = \hat{\delta}_2^2 \neq 0$ ,

$$\sqrt{T} \left[ \hat{\delta}_1^2 - \hat{\delta}_2^2 - (\delta_1^2 - \delta_2^2) \right] \xrightarrow{A} N(0, v_d) \quad (36)$$

where  $v_d = \sum_{\tau=-\infty}^{\infty} E(d_t d_{t-\tau}')$ . In the empirical application, this expression can be

approximated using the well-known Newey–West (1987) estimator given by,

$$\hat{v}_d = \sum_{\tau=-k}^k \left( \frac{k-|\tau|}{k} \right) \frac{1}{T} \sum_{t=1}^T (d_t d_{t-\tau}')$$

## 6.2 GMM Empirical Results

The empirical results using the GMM framework described above and the 20 *sVRP* beta-sorted portfolios are reported in Table 5. Panel A contains the results of the SDF specifications given by models 1 to 4 under both the aggregate consumption growth of non-durable goods and services (NDC) and stockholder growth consumption growth (SHC). The last column of Table 5 displays the HJ distance given by expression (35) with the corresponding *p*-value in parentheses. All alternative specifications are rejected. At the same, the estimators of the preference parameters across models tend to be estimated with a lot of noise. For all preference estimators, standard errors are reported in parentheses.<sup>12</sup>

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<sup>12</sup> In all cases, we check the shape of the objective function when we minimize the weighted average pricing errors according to expression (31) for the parameters estimated under the power, recursive, and



Regarding recursive preferences and power utility and for stockholder consumption, with the exception of recursive preferences with the market return as the proxy for continuation value, the magnitudes and the sign of the risk aversion coefficients are systematically reasonable. For recursive preferences with the market *sVRP* as the continuation value, the risk aversion coefficient is equal to 10.14. Unfortunately, in this case, the sign of the elasticity of intertemporal substitution is negative. A systematic difference when using one approximation of the continuation value or another relies on the sign of the elasticity of intertemporal substitution. When we employ either aggregate consumption growth or stockholder consumption growth and market wealth, the signs of the elasticity of intertemporal substitution are positive and less than one. However, when we use market volatility swaps, the elasticity of intertemporal substitution becomes negative for both types of consumption growth.

We also report the results using the habit preferences for both types of consumption. It is important to notice that the empirical implementation of the model described by equations (11) to (15) simultaneously estimates all preference parameters and the surplus consumption process. To provide some intuition about the behavior of the resulting time-varying risk aversion given by  $\hat{\gamma}/S_t$ , where the curvature parameter estimator is reported in Table 5 and the surplus consumption is obtained using equations (13) and (14), Figure 2 displays the market volatility risk premium and the two-month lagged changes of risk aversion.<sup>13</sup> We observe how the behavior of risk aversion changes follows the previously available payoffs of volatility swaps. Indeed, the correlation coefficient between both series is as large as 0.47. In any case, under the habit preference models, risk aversion estimates are 2.46 and 2.22 for aggregate

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habit preference specifications. The minimum value of the functions corresponds to the parameter estimators reported in Panel A of Table 5. The results strongly suggest that the numbers reported are robust to a large number of alternative initial conditions.

<sup>13</sup> This figure is constructed using stockholder consumption growth.

consumption growth and stockholder consumption growth, respectively, but the estimated coefficients are not statistically different from zero. In addition, the average pricing errors are statistically different from zero and the pricing specification is rejected with a  $p$ -value of the HJ distance of 0.0101.

Panel B of Table 5 contains the results of the linear SDF specifications given by models 5 to 7. As in all previously analyzed models, the linear specifications are rejected. The parameters across the specifications using either the market  $sVRP$  as a factor or the SDF with skewness and kurtosis are estimated with low precision. The average pricing errors are all negative and statistically different from zero. Interestingly, the slope parameters of the two-factor model with the market  $sVRP$  and default are negative and statistically different from zero, which suggests that the risk premia associated with both risk factors are positive and statistically significant.

We next empirically investigate whether competing models exhibit significantly different sample HJ distances. If our alternative specifications fail to find differences in significance across models, it would imply that the proposed factors are too noisy to explain the cross-sectional differences and to conclude that one model is superior to the others. We therefore employ the test statistic given by equation (36) based on the differences between the square of the HJ distances for two given models. Table 6 reports the empirical results. The numbers in this table represent pairwise tests of equality of the squared HJ distances for all alternative specifications of SDF linear and non-linear models. We report the differences between the sample squared HJ distances of the models in row  $i$  and column  $j$ , or  $\hat{\delta}_i^2 - \hat{\delta}_j^2$ . For example, given that the HJ distance of the power aggregate consumption model from Panel A of Table 5 is 0.7078 and the HJ distance for the same model with stockholder consumption is 0.7060, the first number in the first row of Table 6, which is equal to  $0.0026$ , is obtained as  $0.7078^2$

– 0.7060<sup>2</sup>. As discussed above, the asymptotic distribution of this test statistic allows for misspecification of the models. The associated  $p$ -values are provided in parentheses.

The results suggest that, generally, there is not statistical significance between the competing models when we employ the HJ distance. The only important exception is the model that combines the market volatility risk premium and the default premium as factors.<sup>14</sup> The linear SDF on the market volatility risk premium and default premium is statistically superior to all the other models, with the exception of the recursive preference specification using aggregate consumption growth and with either market wealth or the market  $sVRP$  as continuation values.

### 6.3 Two-Pass Cross-Sectional Estimation and Tests

A test of the competing asset pricing models of the determinants of the cross section of volatility risk premia using the models' beta specifications may help clarify matters. In particular, we now test the models described below using our 20  $sVRP$  beta-sorted portfolios. In all cases,  $\lambda_0$  is the zero-beta rate and  $\lambda_k$  for  $k = 1, \dots, K$  are the risk premia associated with the  $K$  aggregate risk factors that drive the cross-sectional variation among volatility swap payoffs for our set of 20 portfolios,  $p = 1, \dots, 20$ , as follows,

a) Model 1: power utility with both aggregate consumption and stockholder consumption:

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{ndc}\beta_c^p \quad (37a)$$

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{shc}\beta_{sc}^p \quad (37b)$$

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<sup>14</sup> A model that recognizes the skewness and kurtosis of the underlying market return is also statistically superior to the one-factor model with the market  $sVRP$ .

b) Model 2: recursive utility with both aggregate consumption and stockholder consumption and market wealth and the market volatility risk premium:

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{ndc}\beta_c^p + \lambda_m\beta_m^p \quad (38a)$$

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{shc}\beta_{sc}^p + \lambda_m\beta_m^p \quad (38b)$$

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{ndc}\beta_c^p + \lambda_{svrp}^m\beta_{msvrv}^p \quad (38c)$$

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{shc}\beta_{sc}^p + \lambda_{svrp}^m\beta_{msvrv}^p \quad (38d)$$

c) Model 3: habit preferences with time-varying risk aversion:

Using the expression of risk aversion under the habit preference model, we can write the consumption surplus as  $S_t = \gamma/RA_t$ , where  $RA_t$  is the time-varying risk aversion. Then, by taking logarithms in expression (12), the SDF can be written as

$$\ln(M_{t,t+\tau}) = e^{\ln \rho - \gamma \Delta c_{t+\tau} + \gamma \Delta ra_{t+\tau}} \cong 1 + \ln \rho - \gamma \Delta c_{t+\tau} + \gamma \Delta ra_{t+\tau} \quad (39)$$

which we write as a beta factor model,

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{ndc}\beta_c^p + \lambda_{ra}\beta_{ra}^p \quad (40a)$$

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{shc}\beta_{sc}^p + \lambda_{ra}\beta_{ra}^p \quad (40b)$$

d) Model 4: the CAPM with market wealth:

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_m\beta_m^p \quad (41)$$

e) Model 5: the Bakshi–Madan (2006) model with higher-order moments:

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_m\beta_m^p + \lambda_{skku}\beta_{m^2}^p \quad (42)$$

d) Model 6: the CAPM with the market volatility risk premium as the only risk factor:

$$E\left(sVRP_{t,t+\tau}^p\right) = \lambda_0 + \lambda_{svrp}^m\beta_{msvrv}^p \quad (43)$$

e) Model 7: a two-factor model with the market volatility risk premium and the default premium:

$$E(sVRP_{t,t+\tau}^P) = \lambda_0 + \lambda_{svrp}^m \beta_{msvrp}^P + \lambda_{def} \beta_{def}^P \quad (44)$$

f) Model 8: a three-factor model with the market volatility risk premium, the default premium, and the HML Fama–French factor:

$$E(sVRP_{t,t+\tau}^P) = \lambda_0 + \lambda_{svrp}^m \beta_{msvrp}^P + \lambda_{def} \beta_{def}^P + \lambda_{hml} \beta_{hml}^P \quad (45)$$

g) Model 9: a four-factor model with the market volatility risk premium, the default premium, the HML Fama–French factor, and market wealth:

$$E(sVRP_{t,t+\tau}^P) = \lambda_0 + \lambda_{svrp}^m \beta_{msvrp}^P + \lambda_{def} \beta_{def}^P + \lambda_{hml} \beta_{hml}^P + \lambda_m \beta_m^P \quad (46)$$

Therefore, we now test the linear versions of the models using the alternative  $K$ -factor beta specifications described above in which the volatility risk premia are linear in the volatility risk premium betas, that is,  $E(sVRP) = X\lambda$ , where  $X = [I_N, \beta]$  and

$\lambda = [\lambda_0, \lambda_1']$  is a vector consisting of the zero-beta rate,  $\lambda_0$ , and the risk premia on the

$K$  factors,  $\lambda_1$ . The pricing errors of the  $N$  portfolios are given by

$$e = E(sVRP) - X\lambda \quad (47)$$

As a goodness-of-fit measure of the competing models, we employ the cross-sectional  $R^2$  defined by Kan, Robotti, and Shanken (2013, KRS hereafter) as

$$R^2 = 1 - \frac{Q}{Q_0} \quad (48)$$

where the  $Q$  statistic given by

$$Q = e'V^{-1}e = E(sVRP)'V^{-1}E(sVRP) - E(sVRP)'V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}E(sVRP)$$

represents the aggregate pricing errors and  $Q_0 = e_0'V^{-1}e_0$  denotes the deviations of the mean returns from their cross-sectional average, with

$$e_0 = \left[ I_N - I_N \left( I_N' V^{-1} I_N \right)^{-1} I_N' V^{-1} \right] E(sVRP)$$

and  $V$  is the variance–covariance matrix of the portfolio volatility risk premia. As KRS point out, the  $R^2$  statistics given by (48) is a relative measure of the goodness- of- fit since it compares the magnitude of the model’s expected return deviations to that of typical deviations from the average expected return. Moreover,  $0 \leq R^2 \leq 1$  and  $R^2$  is a decreasing function of the aggregate pricing errors  $Q$ . Thus,  $R^2$  given by (48) is a reasonable and well-defined measure of goodness- of- fit. Note that, in fact, we employ  $R^2$  for average returns rather than the average of monthly  $R^2$  values.

In addition, KRS show how to perform a test of whether the model has any explanatory power for pricing assets cross-sectionally. In other words, they test whether we can reject the null hypothesis of  $R^2 = 0$ . The asymptotic test is given by

$$TR^2 \xrightarrow{A} \sum_{i=1}^K \frac{\xi_i}{Q_0} x_i \quad (49)$$

where the  $x_i$ 's are independent  $\chi^2(1)$  random variables and the  $\xi_i$ 's are the  $K$  nonzero eigenvalues of

$$\left[ \beta' V^{-1} \beta - \beta' V^{-1} I_N \left( I_N' V^{-1} I_N \right)^{-1} I_N' V^{-1} \beta \right] \text{Var}(\hat{\lambda}_I)$$

where  $\text{Var}(\hat{\lambda}_I)$  is the expression adjusted by errors-in-the-variable and misspecification of the model.<sup>15</sup> In particular, the asymptotic distribution of  $\hat{\lambda}$  under the misspecified models is

$$\sqrt{T}(\hat{\lambda} - \lambda) \xrightarrow{A} N(0_{K+1}, \text{Var}(\hat{\lambda})) \quad (50)$$

where  $\text{Var}(\hat{\lambda}) = \sum_{\tau=-\infty}^{\infty} E(v_t v_{t-\tau}')$  and

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<sup>15</sup> The  $p$ -values to test the null  $H_0 : R^2 = 0$  are calculated as before, using the procedure of Jagannathan and Wang (1996), HJ and Parker and Julliard (2005).

$$v_t = \underbrace{(\lambda_t - \lambda)}_{\text{var when true betas}} - \underbrace{(\eta_t - \eta)\omega_t}_{\text{EIV adjustment}} + \underbrace{H z_t u_t}_{\text{misspecification term}} \quad (51)$$

with

$$\eta_t = [\lambda_{0,t}, (\lambda_{1,t} - f_t)']'; \eta = [\lambda_0, (\lambda_1 - f)']'; u_t = e'V^{-1}(sVRP_t - E(sVRP))$$

$$\omega_t = \lambda_1' \Omega^{-1}(f_t - f); z_t = [0, (f_t - f)' \Omega^{-1}]'$$

$$H = (X'V^{-1}X)^{-1}$$

where  $\Omega$  is the variance–covariance matrix of the factors denoted by  $f_t$ .

Finally, we present the test for comparing two competing models. Suppose

$M_1 \neq M_2$  and  $0 < R_1^2 = R_2^2 < 1$ . Then

$$\sqrt{T}(\hat{R}_1^2 - \hat{R}_2^2) \xrightarrow{A} N\left(0, \sum_{\tau=-\infty}^{\infty} E(d_t d_{t-\tau})\right) \quad (52)$$

where

$$d_t = Q_0^{-1} \left( u_{1t}^2 - 2u_{1t}M_{1t} - u_{2t}^2 + 2u_{2t}M_{2t} \right)$$

$$u_{1t} = e_1'V^{-1}(sVRP_t - E(sVRP)) \quad \text{and} \quad u_{2t} = e_2'V^{-1}(sVRP_t - E(sVRP))$$

#### 6.4 Two-Pass Cross-Sectional Empirical Results

As in Section 6.3, Panel A of Table 7 contains the results of the two-pass cross-sectional regressions using consumption-based factors, while Panel B of Table 7 displays the results concerning factor-based models.

In all cases, we adapt the testing framework discussed above to the Fama–MacBeth (1973) two-pass cross-sectional methodology, where we estimate rolling betas using the first 60 months of the sample as a fixed estimation period and then use a rolling window of 59 months of past data plus the month in which we perform the cross-sectional regression with the 20 portfolios. Hence, for each month  $t$  we always

employ a beta estimated with 60 observations. Moreover, below all risk premia estimators, we report the  $p$ -values associated with the traditional Fama–MacBeth standard error in parentheses and in brackets, with the standard error adjusted for errors in variables, and the potential misspecification of the model as captured by expression (51). We also provide two measures of goodness-of-fit. We report the mean absolute pricing error ( $MAE$ ) calculated as

$$MAE = \frac{1}{20} \sum_{p=1}^{20} \left| \bar{\tilde{e}}_p \right| \quad (53)$$

where  $\bar{\tilde{e}}_p$  is the mean pricing error associated with each of the 20 portfolios. The last column of Table 7 reports the  $\hat{R}^2$  value given by equation (48), where below we display the  $p$ -value for the test of the null hypothesis given by  $R^2 = 0$  from expression (49) and in brackets we report the standard error of  $\hat{R}^2$  under the assumption that  $0 \leq R^2 \leq 1$ .

Regarding consumption models, the results suggest that the standard errors of the risk premia estimators are very sensitive to potential model misspecification. At the same time, in most cases, the estimator of the zero-beta rate is statistically different from zero independently of the adjustment. These results already put into question the validity of the models. Indeed, all risk premia associated with consumption growth, either aggregate consumption or stockholder consumption, are not statistically different from zero. Consumption risk does not seem to be priced in the cross section of the volatility risk premia. The only statistically significant risk premia are the market portfolio return in the case of the recursive preference model with aggregate consumption growth and that related to the market volatility risk premium in the recursive model when we approximate the continuation value with volatility swaps rather than with the market portfolio return. As theory suggests, the sign of the statistically significant risk premium associated with market wealth is positive and it



becomes negative when we employ the market *sVRP* under recursive preferences. For habit preferences, the risk premium is negatively related to changes in risk aversion, with a *p*-value of 0.096, when we employ stockholder consumption growth but it lacks a lot of precision when we use aggregate consumption.<sup>16</sup> Two additional results of Panel A are relevant. First, the *MAE* values reported in Panel A tend to be higher than those of Panel B. Second, for all models, we cannot reject the null hypothesis that  $R^2$  is statistically equal to zero, since the standard errors of the  $\hat{R}^2$  values suggest all models are estimated with a great deal of noise.

It may be easily the case that consumption risk is able to explain the cross section of volatility risk premia as long as we introduce ambiguity in the SDF. Under ambiguity aversion, Miao, Wei, and Zhou (2012) show that the market variance premium can be generated without resorting to exogenous stochastic volatility or jumps. By calibrating their model, they conclude that 96% of the market variance risk premium can be attributed to ambiguity aversion. Unfortunately, it is not clear how their approach can be extended to test the models cross-sectionally and with market data.

Panel B of Table 7 shows that factor-based models explain the cross section of volatility risk premia much more accurately. In three cases, the asset pricing specification is not statistically rejected. These models always include the market *sVRP* and the default premium. They are also the models with a lower *MAE*. It is also true that these models with HML and the excess market returns as additional factors are not rejected, but the coefficients associated with either the excess market return or the HML factor are not statistically different from zero. However, in all three cases, the market

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<sup>16</sup> Understanding volatility swaps as hedging products, we theoretically expect a negative coefficient when regressing volatility risk premia on risk aversion betas as in expression (40).

volatility risk premium beta is significantly priced, with the expected negative sign.<sup>17</sup> Again, in all three cases, the default risk premium beta is positive and statistically different from zero. Hence, the higher the default beta, the higher the average payoff expected from volatility swaps in the cross section. Therefore, we find that, on average, the market *sVRP* is priced across portfolios and investors are compensated for bearing credit (default) risk. The two-factor model for volatility risk generates statistically significant risk premia of -0.006 and 0.012 for market volatility risk and default risk, respectively. The  $\hat{R}^2$  of the two-factor model is equal to 0.514 and is statistically different from zero, with a standard error of 0.211.<sup>18</sup> Figure 3 displays the average realized *sVRP* against the fitted value for a selection of asset pricing models. The two-factor model presents a better visual fit across all models. In any case, the difficulty of the theoretical two-factor model in explaining portfolio P20B must be recognized. The model generates a negative payoff for this portfolio, which is too extreme (too highly negative) to obtain a more precise linear fit relative to actual data.

Finally, Table 8 contains the pairwise tests of equality of the two-pass cross sectional regression  $R^2$  values for alternative factor pricing models using the 20 *sVRP* beta-sorted portfolios. It contains the pairwise tests of equality of the two-pass cross-sectional regression  $R^2$  values for alternative factor pricing models. We report the difference between the sample cross-sectional  $R^2$  values of the models in row  $i$  and column  $j$ ,  $\hat{R}_i^2 - \hat{R}_j^2$ , and the associated  $p$ -values in parentheses for the test of  $\hat{R}_i^2 = \hat{R}_j^2$ .

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<sup>17</sup> The negative sign reflects the fact that the market volatility risk premium tends to be positive in events of high marginal utility.

<sup>18</sup> Similar results are found when we estimate the two-pass cross-sectional regression using a constant beta throughout the sample period. Moreover, when we check all our empirical results using the alternative set of 20 portfolios ranked according to the level of the volatility risk premium, the results are qualitative the same independently of using consumption-based models or factor-based specifications. The two-factor model, with the market volatility risk premium and the default premium, presents a better competing performance than the rest of the models analyzed in our research. All empirical results are available upon request from the authors.

As before, these  $p$ -values allow for misspecifications of the models. The role of the default premium seems to be important for the cross-sectional pricing of volatility swaps, even under a statistical comparison of  $R^2$  values. However, we cannot reject that the  $\hat{R}^2$  values between the two-factor model and the model extended with the HML factor and the excess market returns are equal. On the other hand, the two-factor model performs relatively well when compared with competing models. In any case, the results make clear the difficulty of distinguishing between the models from a statistical point of view. For example, the power of the test seems to be low when we only incorporate consumption data in the models or when we compare the two-factor model with consumption-based specifications. These models are estimated with a considerable amount of noise. We should not simply compare the point estimates of the  $\hat{R}^2$  values. As pointed out by KRS, it seems reasonable to focus on individual  $\hat{R}^2$  values rather than on differences across models.

## **7. Why Does the Default Premium Explain the Cross sectional Variation of Volatility Risk Premia?**

The default beta risk with respect to the volatility risk premia seems to be consistently priced in our cross section. We next provide an intuitive but rigorous explanation of this finding. We employ the underlying components of the 20  $sVRP$  beta-sorted portfolios to construct the corresponding 20 return portfolios. The first column of Table 9 reports the results of regressing the rate of returns of the 20 portfolios on the market return and the default premium. We display the default return beta once we control for the market return. Similarly, the second column contains the default return beta, as before controlling for the market return, but now with respect to the St. Louis Fed Financial Stress Index (STLFSI). The STLFSI measures the degree of financial stress in the

markets and is constructed from 18 series: seven interest rate series, six yield spreads, and five other indicators. Each of these variables captures some aspect of financial stress. In this regard, it is a broader measure of financial credit risk or financial stress than the default premium. By construction, the average value of the index is equal to zero. Thus, zero reflects normal financial market conditions. Values below zero suggest below-average financial stress, while values above zero indicate above-average financial stress.<sup>19</sup> Increasing values of this index can therefore be interpreted in the same way as increasing values of the default premium.

The empirical results from the first two columns of Table 9 suggest a similar interpretation. The behavior of the components of portfolios P1B, P2B and P3B is very different from the behavior of the underlying components of portfolios P19B and P20B. Recall that the first portfolios have, on average, positive volatility risk premia, while the last two portfolios have negative average volatility risk premia. Using either the default premium or the STLFSI, the relation between the returns of the first three portfolios and financial stress is positive. When default or the financial stress index increases, the returns of these portfolios increase. These portfolios seem to be good hedgers relative to financial stress. However, the last two portfolios move negatively with respect to financial stress. Even when the market return is controlled for, when measures of financial stress increase, their return significantly decreases. These results suggest that investors may rationally hedge the financial stress risk of these components by buying volatility swaps. For those assets negatively affected by financial stress, they are willing to pay a high volatility swap to cover that credit/financial risk stress. Therefore, on average, we can expect a negative payoff from holding long positions on volatility swaps associated with these assets and a positive average payoff from assets moving

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<sup>19</sup> See <http://www.stlouisfed.org/newsroom/financial-stress-index/> for further details.

positively with default risk. This is exactly what we display in Table 9. It seems that the differences in the cross section of our *sVRP* beta-sorted portfolios reflect a very different behavior of these assets with respect to credit/financial stress. To complete our argument, we should find evidence that the volatility risk premia of portfolios P19B and P20B move positively with financial stress. In other words, the volatility payoff of these portfolios should cover increasing financial stress risk. This is again what we report in the third column of Table 9.

A second possibility to justify the pricing of default betas in the cross section of the volatility risk premia is to replicate the findings of Frazzini and Pedersen (2014) with our data and sample period. A well-known empirical finding in asset pricing is that the relation between average returns and beta risk is too flat relative to the theoretical predictions of the CAPM. Frazzini and Pedersen (2014) argue that an asset pricing model with leverage and margin constraints is able to explain this anomaly. By leveraging and de-leveraging the tangency portfolio, investors can control their risk–return tradeoff according to their risk preferences. However, some institutional investors cannot use leverage and other investors who are able to employ leverage are constrained by their margin requirements. These investors will overweight risky assets implying that high-beta assets require lower risk-adjusted returns than low-beta assets. The authors illustrate their argument by proposing a market neutral betting-against-beta (BAB) factor consisting of long levered low-beta stocks and short de-levered high-beta securities:

$$R_{t+1}^{BAB} = \frac{1}{\beta_t^L} (R_{t+1}^L - R_f) - \frac{1}{\beta_t^H} (R_{t+1}^H - R_f) \quad (54)$$

where  $L$  and  $H$  represent low- and high- beta respectively. The authors provide convincing evidence that the BAB generates high and consistent performance in each of the major global markets and asset classes and that the results are independent of the

asset pricing model employed. A key result is that when funding constraints become more binding and the leveraged investors hit their margin constraint, they must de-leverage. This suggests the required rate of return of portfolio BAB increases and the contemporaneous realized BAB returns tend to become negative.

Using the rates of return of our 20 *sVRP* beta-sorted portfolios, and our sample period, we construct the BAB factor using expression (54). Table 10 contains the alphas generated by our BAB factor when we control for typical asset pricing risk factors. In particular, we regress the BAB factor returns on the market, on the Fama–French factors, and on the three-factor model augmented with the momentum factor and the aggregate liquidity measure of Pastor and Stambaugh (2003). As expected, the BAB portfolio consistently shows positive and statistically significant risk-adjusted returns. However, when we control for the market excess return and either funding liquidity, captured by the TED spread, or credit risk proxied by the default premium, the alphas are no longer statistically significant. This suggests that the tightening of funding liquidity and borrowing constraints may explain the behavior of the extreme *sVRP* beta-sorted portfolios in terms of average volatility risk premia and their betas. As in the case of the Frazzini and Pedersen (2014) paper, funding liquidity seems to have important implications for asset pricing and, in particular, for pricing volatility swaps.

## **8. Conclusions**

Most of the literature dealing with variance or volatility swaps is concerned with the variance risk premium at the market level. The empirical evidence shows that the market variance risk premium has very useful economic information content. Given this evidence, it is surprising how little research analyzes variance or volatility swaps at the individual or portfolio level. This paper discusses and tests the cross-sectional variation

of the volatility risk premia for a set of 20 portfolios. We rank individual  $sVRP$  values by their betas with respect to the market volatility risk premium. Accordingly, we employ a set of 20  $sVRP$  beta-sorted portfolios to analyze the determinants of their cross-sectional variation. We show that beta with respect to the market volatility risk premia and the default beta have statistically significant risk premia that help explaining the cross-sectional variation of average volatility risk premia. This is especially the case for the default premium factor and the empirical result holds even if we allow for potential misspecification of the models. Finally, we relate our findings to credit/financial stress risk and to funding liquidity risk. We show that the success of the default premium in the cross-sectional variation of the volatility risk premia can be explained by the very different behavior that the underlying components of our 20  $sVRP$  beta-sorted portfolios have with respect to financial stress risk.

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Table 1  
Volatility Risk Premia: Descriptive Statistics and Betas,  
Portfolios Sorted by Volatility Risk Premium Betas, January 1996 to February 2011

<i>sVRP</i> Beta- Sorted Portfolios	Average <i>sVRP</i> (Monthly)	Average <i>sVRP</i> (Daily)	Standard Deviation (Monthly)	Standard Deviation (Daily)	<i>sVRP</i> Beta (S&P100 Market <i>sVRP</i> )	Market Return Beta (Overall US Market)	Market Return Beta (S&P100 Market)	Relative Bid- Ask Spread
P1B	0.103	0.101	0.179	0.188	-0.946	1.164	1.168	0.257
P2B	0.040	0.043	0.092	0.096	-0.229	1.042	1.050	0.256
P3B	0.024	0.023	0.082	0.080	0.056	0.893	0.922	0.259
P4B	0.018	0.014	0.072	0.067	0.223	1.017	1.008	0.265
P5B	0.009	0.005	0.066	0.061	0.307	0.758	0.771	0.260
P6B	0.001	-0.002	0.062	0.060	0.368	0.890	0.897	0.270
P7B	-0.0002	-0.006	0.067	0.063	0.511	0.884	0.918	0.268
P8B	-0.004	-0.010	0.067	0.063	0.558	0.964	0.987	0.261
P9B	-0.010	-0.016	0.069	0.065	0.704	0.850	0.851	0.270
P10B	-0.010	-0.017	0.077	0.073	0.819	0.931	0.977	0.273
P11B	-0.019	-0.023	0.079	0.076	0.919	0.867	0.868	0.269
P12B	-0.021	-0.027	0.086	0.083	1.011	0.949	0.958	0.281
P13B	-0.026	-0.030	0.088	0.090	1.009	0.823	0.874	0.275
P14B	-0.022	-0.032	0.099	0.099	1.219	0.972	1.013	0.279
P15B	-0.028	-0.034	0.106	0.111	1.327	1.012	1.020	0.278
P16B	-0.031	-0.037	0.119	0.125	1.444	0.873	0.935	0.277
P17B	-0.029	-0.039	0.139	0.139	1.782	1.138	1.142	0.283
P18B	-0.029	-0.043	0.165	0.162	2.068	1.163	1.164	0.281
P19B	-0.035	-0.046	0.192	0.193	2.420	1.233	1.241	0.286
P20B	-0.034	-0.045	0.312	0.318	3.891	1.463	1.521	0.296
Market <i>sVRP</i>	-0.014	-0.014	0.069	0.069	1.000	0.929	1.000	-

The volatility risk premium (*sVRP*) for each portfolio is defined as the difference between the realized volatility and the model-free risk-neutral integrated return volatility over the corresponding month. The risk-neutral volatility is obtained by the set of prices of options on each underlying individual security with one month to maturity. The numbers reported are the annualized volatility risk premia for both the 20 portfolios and the S&P 100 Index. Portfolio 1 contains the securities with the lowest *sVRP* betas and portfolio 20 includes securities with the highest *sVRP* betas. The portfolios are updated each month during the sample period. The *sVRP* beta is the OLS regression coefficient from linear regressions of the monthly *sVRP* of each portfolio on the *sVRP* of the S&P 100 market index. The market return betas are the OLS regression coefficients from linear regressions of the monthly return of each portfolio on the market return index given by either the S&P 100 Index or the overall US value-weight market return of all CRSP firms listed on the NYSE, AMEX, or NASDAQ. The monthly data refers to the observation of each portfolio on the last day of each month. The betas are always estimated at the monthly frequency. The relative bid-ask spread is the average bid-ask spread for all traded options on the underlying stock that belong to a given portfolio calculated at the end of the last day of each month.

Table 2  
Correlation Coefficients between the Volatility Risk Premia for Representative *sVRP* Beta-Sorted Portfolios, January 1996 to February 2011

Panel A: Monthly Correlations	P5B	P10B	P15B	P20B	Market <i>sVRP</i>
P1B	0.414	-0.152	-0.381	-0.452	-0.366
P5B	1	0.607	0.314	0.194	0.323
P10B		1	0.834	0.726	0.736
P15B			1	0.927	0.863
P20B				1	0.863
Panel B: Daily Correlations	P5B	P10B	P15B	P20B	Market <i>sVRP</i>
P1B	0.427	-0.183	-0.441	-0.538	-0.435
P5B	1	0.589	0.333	0.155	0.231
P10B		1	0.865	0.685	0.733
P15B			1	0.911	0.828
P20B				1	0.841

This table reports the correlation coefficients estimated for the overall sample period using monthly (daily) data for the volatility risk premia of the representative portfolios. The volatility risk premium (*sVRP*) for each portfolio is defined as the difference between the realized volatility and the model-free risk-neutral integrated return volatility over the corresponding month. The risk-neutral volatility is obtained by the set of prices of options on each underlying individual security with one month to maturity. Portfolio 1 contains the securities with the lowest *sVRP* betas, and portfolio 20 includes securities with the highest *sVRP* betas. The portfolios are updated each month during the sample period.

Table 3  
Correlation Coefficients between State Variables, January 1996 to February 2011

Monthly Correlations	Excess US Market Return	Cons Growth	Stockholder Cons Growth	DEF	SMB	HML	MOM
Market <i>sVRP</i>	-0.273	-0.189	-0.118	0.075	0.019	0.130	0.185
Excess Market Return	1	0.213	0.769	-0.132	0.242	-0.247	-0.296
Cons Growth		1	0.131	-0.356	0.043	-0.125	-0.356
Stockholder Cons Growth			1	-0.149	0.449	0.237	-0.301
DEF				1	0.058	-0.087	-0.198
SMB					1	-0.372	0.091
HML						1	-0.156

This table reports the correlation coefficients estimated for the overall sample period using monthly data. The market volatility risk premium is defined as the difference between the realized volatility and the model-free risk-neutral integrated return volatility over the corresponding month. The risk-neutral volatility is obtained by the set of prices of options on the S&P100 index with one month to maturity. In this table, Cons Growth indicates the monthly growth rate of seasonally adjusted real per capita consumption expenditures on non-durables goods and services; Stockholder Cons Growth is the Malloy, Moskowitz, and Vissing-Jorgensen (2011) measure of consumption growth from stockholders; Excess Market Return, SMB, HML, and MOM are the Fama-French factors, and the momentum factor obtained from the Kenneth French's website, and DEF is the default premium calculated as the difference between Moody's yield on Baa corporate Bbonds and the 10-year government bond yield.

Table 4

Panel A: Consumption and Market Factor Betas for Five Portfolios Sorted by the Volatility Risk Premium Beta, January 1996 to February 2011

<i>sVRP</i> Beta-Sorted Portfolios	Market <i>sVRP</i>	Market <i>sVRP</i>	Excess Market Return	Cons Growth	Market <i>sVRP</i>	Excess Market Return	Stock Cons Growth
P1B Beta (t-stat) [R <sup>2</sup> -adj]	-0.946 (-5.28) [0.129]	-0.764 (-4.18) [0.178]	0.257 (3.43)	0.440 (0.31)	-0.757 (-4.14) [0.178]	0.307 (2.65)	-0.255 (-0.51)
P5B Beta (t-stat) [R <sup>2</sup> -adj]	0.307 (4.58) [0.100]	0.402 (6.04) [0.193]	0.116 (4.25)	0.748 (1.43)	0.386 (5.76) [0.184]	0.113 (2.66)	0.054 (0.30)
P10B Beta (t-stat) [R <sup>2</sup> -adj]	0.819 (14.61) [0.540]	0.873 (15.37) [0.571]	0.026 (1.11)	1.561 (3.49)	0.844 (14.35) [0.542]	0.033 (0.89)	0.037 (0.23)
P15B Beta (t-stat) [R <sup>2</sup> -adj]	1.327 (22.94) [0.744]	1.327 (22.45) [0.757]	-0.050 (-2.08)	1.429 (3.07)	1.294 (21.35) [0.745]	-0.063 (-1.63)	0.138 (0.84)
P20B Beta (t-stat) [R <sup>2</sup> -adj]	3.891 (22.87) [0.743]	3.769 (21.58) [0.754]	-0.227 (-3.17)	1.292 (0.94)	3.706 (21.28) [0.756]	-0.347 (-3.15)	0.732 (1.55)

Panel B: Default Premium, Consumption, and Market Factor Betas for Five Portfolios Sorted by the Volatility Risk Premium Beta, January 1996 to February 2011

<i>sVRP</i> Beta-Sorted Portfolios	Market <i>sVRP</i>	Market <i>sVRP</i>	Excess Market Return	DEF	Market <i>sVRP</i>	Cons Growth	DEF
P1B Beta (t-stat) [R <sup>2</sup> -adj]	-0.946 (-5.28) [0.129]	-0.777 (-4.30) [0.180]	0.267 (3.60)	0.296 (0.75)	-0.917 (-5.02) [0.126]	1.648 (1.06)	0.290 (0.67)
P5B Beta (t-stat) [R <sup>2</sup> -adj]	0.307 (4.58) [0.100]	0.393 (5.98) [0.196]	0.117 (4.34)	-0.242 (-1.69)	0.334 (4.97) [0.122]	0.834 (1.45)	-0.234 (-1.47)
P10B Beta (t-stat) [R <sup>2</sup> -adj]	0.819 (14.61) [0.540]	0.856 (15.48) [0.587]	0.028 (1.25)	-0.531 (-4.40)	0.860 (16.06) [0.596]	1.088 (2.38)	-0.443 (-3.50)
P15B Beta (t-stat) [R <sup>2</sup> -adj]	1.327 (22.94) [0.744]	1.309 (22.37) [0.758]	-0.046 (-1.94)	-0.402 (-3.15)	1.358 (23.69) [0.757]	0.903 (1.85)	-0.286 (-2.11)
P20B Beta (t-stat) [R <sup>2</sup> -adj]	3.891 (22.87) [0.743]	3.747 (21.60) [0.753]	-0.217 (-3.04)	-0.053 (-0.14)	3.903 (22.42) [0.740]	0.739 (0.50)	0.152 (0.37)

This table reports the OLS risk premium volatility betas. The volatility risk premium (*sVRP*) for each portfolio is defined as the difference between the realized volatility and the model-free risk-neutral integrated return volatility over the corresponding month. The risk-neutral volatility is obtained by the set of prices of options on each underlying individual security with one month to maturity. Portfolio 1 contains the securities with the lowest *sVRP* betas and portfolio 20 includes securities with the highest *sVRP* betas. The portfolios are updated each month during the sample period. The *sVRP* beta is the OLS regression coefficient from linear regressions of the monthly *sVRP* of each portfolio on the *sVRP* of the S&P 100 market index, consumption growth, stockholder consumption growth, the US stock market return, and the default premium. The monthly data refers to the observation of each portfolio on the last day of each month. The betas are always estimated at the monthly frequency.

Table 5  
GMM Estimation for Alternative Volatility Risk Premium Models Using  
Portfolios Sorted by the Volatility Risk Premium Betas, January 1996 to February 2011

Panel A		$\gamma$	$\alpha$	$\kappa$	$a$	$b$	$c$	HJ Distance
Power	NDC	-34.873 (106.43)	-0.0023 (0.0012)	-	-	-	-	0.7078 (0.0000)
	SHC	7.810 (12.61)	-0.0027 (0.0010)	-	-	-	-	0.7060 (0.0000)
Recursive	NDC	-372.454 (166.28)	-0.0007 (0.0017)	75.601 (65.14)	-	-	-	0.6956 (0.0009)
	SHC	-7.394 (20.91)	-0.0027 (0.0010)	3.483 (4.26)	-	-	-	0.7018 (0.0000)
Habit	NDC	2.461 (7.40)	-0.0030 (0.0012)	-	-	-	-	0.7812 (0.0007)
	SHC	2.224 (4.86)	-0.0029 (0.0013)	-	-	-	-	0.7644 (0.0101)
Recursive $sVRP^m$	NDC	-308.509 (166.89)	-0.0003 (0.0015)	-41.286 (69.85)	-	-	-	0.7012 (0.0074)
	SHC	10.140 (15.47)	-0.0028 (0.0011)	-40.819 (872.33)	-	-	-	0.7056 (0.0000)
Panel B		$\gamma$	$\alpha$	$\kappa$	$a$	$b$	$c$	HJ-D
Linear $M$ on $R_m + R_m^2$		-	-0.0024 (0.0014)	-	-0.0267 (0.432)	-0.0101 (0.148)	-0.0001 (0.001)	0.6875 (0.0000)
Linear $M$ on $sVRP^m$		-	-0.0025 (0.0010)	-	0.1406 (0.719)	-0.0058 (0.576)	-	0.6994 (0.0000)
Linear $M$ on $sVRP^m + DEF$		-	-0.0032 (0.0008)	-	1.3874 (0.495)	-0.4511 (0.117)	-0.4634 (0.215)	0.5871 (0.0000)

This table reports the parameters obtained under the GMM estimation of alternative asset pricing models with different preference specifications using the second order moments matrix as the weighting GMM matrix for all cases. The numbers in parentheses below the estimated parameters are standard errors while the numbers in parentheses below the HJ distance are  $p$ -values. In this table, NDC refers to non-durable consumption and SHC indicates stockholder aggregate consumption. All models are estimated with monthly data. Habit is the Campbell-Cochrane model, where the estimated gamma is estimated simultaneously with the estimation of the surplus consumption process. The recursive specification under  $sVRP^m$  includes consumption growth and the market volatility risk premium as the second factor rather than the stock market return. The linear SDF specifications include a model that allows for skewness as a determinant factor for volatility risk premia, the market volatility risk premium as the individual factor, and the model adding default as the second factor.



Table 6  
 Model Comparison Using the HJ Distance for Portfolios Sorted by the Volatility Risk Premium Betas:  
 Tests of the Equality of the Squared HJ Distance

Models	Power SHC	Recur NDC	Recur SHC	Habit NDC	Habit SHC	Recur sVRP <sup>m</sup> NDC	Recur sVRP <sup>m</sup> SHC	Linear <i>M on</i> $R_m + R_m^2$	Linear <i>M on</i> sVRP <sup>m</sup>	Linear <i>M on</i> sVRP <sup>m</sup> + DEF
Power NDC	0.0026 (0.718)	0.0172 (0.948)	0.0085 (0.438)	-0.1093 (0.181)	-0.0832 (0.425)	0.0093 (0.962)	0.0032 (0.663)	0.0284 (0.073)	0.0118 (0.540)	0.1563 (0.000)
Power SHC		0.0146 (0.956)	0.0059 (0.477)	-0.1119 (0.162)	-0.0858 (0.390)	0.0068 (0.973)	0.0006 (0.713)	0.0259 (0.142)	0.0093 (0.658)	0.1538 (0.000)
Recur NDC			-0.0087 (0.974)	-0.1265 (0.650)	-0.1004 (0.722)	-0.0078 (0.932)	-0.0140 (0.958)	0.0113 (0.966)	-0.0053 (0.984)	0.1392 (0.605)
Recur SHC				-0.1178 (0.138)	-0.0917 (0.369)	0.0008 (0.997)	-0.0053 (0.572)	0.0200 (0.323)	0.0034 (0.887)	0.1478 (0.000)
Habit NDC					0.0261 (0.838)	0.1186 (0.585)	0.1125 (0.162)	0.1377 (0.111)	0.1211 (0.169)	0.2656 (0.004)
Habit SHC						0.0925 (0.682)	0.0864 (0.385)	0.1117 (0.307)	0.0951 (0.389)	0.2395 (0.032)
Recur sVRP <sup>m</sup> NDC							-0.0061 (0.975)	0.0191 (0.925)	0.0025 (0.990)	0.1470 (0.472)
Recur sVRP <sup>m</sup> SHC								0.0252 (0.147)	0.0086 (0.676)	0.1531 (0.000)
Linear <i>M on</i> $R_m + R_m^2$									-0.0166 (0.000)	0.1279 (0.000)
Linear <i>M on</i> sVRP <sup>m</sup>										0.1445 (0.000)

The reported numbers are the results of pairwise tests of equality of the squared HJ distance for alternative specifications of SDF linear and non-linear models. We report the difference between the sample squared HJ-distances of the modes in row  $i$  and column  $j$ ,  $\hat{\delta}_i^2 - \hat{\delta}_j^2$ , and the associated  $p$ -value in parentheses for the test of the null hypothesis:  $\hat{\delta}_i^2 = \hat{\delta}_j^2$ . The  $p$ -values are computed under the assumption that the models are potentially misspecified.

Table 7

Two-Pass Cross-Sectional Fama–MacBeth Estimation for Alternative Volatility Risk Premium Models,  
Using Portfolios Sorted by the Volatility Risk Premium Betas, January 1996 to February 2011

Panel A: Two-Pass Cross-Sectional Regressions with Consumption-Based Factors												
SDF		$\lambda_0$	$\lambda_{ndc}$	$\lambda_{shc}$	$\lambda_{ra}$	$\lambda_m$	$\lambda_m^2$	$\lambda_{svrp}^m$	$\lambda_{def}$	$\lambda_{hml}$	MAE	$R^2$
Power	NDC	0.001 (0.601) [0.717]	-0.000 (0.902) [0.972]	-	-	-	-	-	-	-	0.0049	0.0057 (0.908) [0.222]
	SHC	0.002 (0.400) [0.370]	-	0.014 (0.000) [0.118]	-	-	-	-	-	-	0.0036	0.0152 (0.759) [0.348]
Recursive	NDC	-0.003 (0.015) [0.356]	0.003 (0.000) [0.510]	-	-	0.064 (0.000) [0.054]	-	-	-	-	0.0029	0.0635 (0.803) [0.369]
	SHC	-0.000 (0.764) [0.763]	-	0.004 (0.039) [0.507]	-	0.031 (0.000) [0.161]	-	-	-	-	0.0033	0.0541 (0.627) [0.163]
Habit	NDC	0.002 (0.165) [0.112]	0.001 (0.216) [0.651]	-	-0.016 (0.303) [0.692]	-	-	-	-	-	0.0035	0.0117 (0.900) [0.421]
	SHC	0.001 (0.536) [0.572]	-	0.008 (0.001) [0.287]	-0.078 (0.000) [0.096]	-	-	-	-	-	0.0028	0.031 (0.824) [0.224]
Recursive $sVRP^m$	NDC	0.002 (0.075) [0.509]	0.002 (0.003) [0.608]	-	-	-	-	-0.007 (0.000) [0.027]	-	-	0.0028	0.1290 (0.417) [0.284]
	SHC	0.002 (0.103) [0.213]	-	0.001 (0.749) [0.931]	-	-	-	-0.007 (0.000) [0.153]	-	-	0.0031	0.0873 (0.462) [0.170]

Panel B: Two-Pass Cross-Sectional Regressions with State Variables-Based Factors												
SDF		$\lambda_0$	$\lambda_{ndc}$	$\lambda_{shc}$	$\lambda_{ra}$	$\lambda_m$	$\lambda_m^2$	$\lambda_{svrp}^m$	$\lambda_{def}$	$\lambda_{hml}$	MAE	$R^2$
$sVRP^m+DEF$ +HML		0.009 (0.000) [0.002]	-	-	-	-	-	-0.007 (0.000) [0.014]	0.012 (0.000) [0.000]	0.021 (0.003) [0.255]	0.0017	0.5233 (0.009) [0.242]
		0.009 (0.000) [0.002]	-	-	-	-0.010 (0.154) [0.658]	-	-0.007 (0.000) [0.049]	0.014 (0.000) [0.000]	0.019 (0.006) [0.335]	0.0016	0.5341 (0.017) [0.219]
CAPM		0.002 (0.284) [0.285]	-	-	-	0.031 (0.001) [0.167]	-	-	-	-	0.0035	0.0751 (0.292) [0.176]
		0.001 (0.550) [0.648]	-	-	-	0.036 (0.000) [0.014]	0.001 (0.041) [0.295]	-	-	-	0.0022	0.1031 (0.193) [0.148]
$sVRP^m$		0.005 (0.000) [0.001]	-	-	-	-	-	-0.006 (0.000) [0.109]	-	-	0.0035	0.0879 (0.148) [0.173]
		0.007 (0.000) [0.006]	-	-	-	-	-	-0.006 (0.000) [0.049]	0.012 (0.000) [0.000]	-	0.0019	0.5139 (0.001) [0.211]

We report the parameter estimated from the two-pass cross sectional regression with rolling betas for alternative asset pricing models. MAE is the mean pricing errors associated with the 20 portfolios ranked by their volatility risk premia. The  $R^2$  value is the sample cross-sectional  $R^2$  as calculated by KRS. The numbers in parentheses are the traditional Fama–MacBeth standard errors of the alternative parameter estimates and the numbers in brackets are  $p$ -values associated with the KRS standard errors adjusted by errors-in-the-variables and potential misspecification of the models. Below the cross-sectional  $R^2$  values, we report the  $p$ -value for the test of  $H_0: R^2 = 0$  and in brackets we display the standard error of  $R^2$  under the assumption that  $0 < R^2 < 1$ .

Table 8  
Model Comparison Using the Two-Pass Cross-Sectional Fama–MacBeth Estimation for Portfolios Sorted  
by the Volatility Risk Premium Betas: Tests of the Equality of the Cross-Sectional  $R^2$  Values

Models	Power SHC	Recur NDC	Recur SHC	Habit NDC	Habit SHC	Recur $sVRP^m$ NDC	Recur $sVRP^m$ SHC	$sVRP^m + DEF + HML$	$sVRP^m + DEF + HML + R_m$	CAPM	$R_m + R_m^2$	$sVRP^m$	$sVRP^m + DEF$	
Power NDC	-0.0096 (0.978)	-0.0579 (0.902)	-0.0484 (0.841)	-0.0060 (0.986)	-0.0256 (0.920)	-0.1234 (0.762)	-0.0816 (0.745)	-0.5176 (0.124)	-0.5284 (0.100)	-0.0694 (0.759)	-0.0975 (0.708)	-0.0822 (0.712)	-0.5083 (0.112)	
Power SHC		-0.0483 (0.894)	-0.0389 (0.891)	0.0036 (0.994)	-0.0160 (0.945)	-0.1138 (0.788)	-0.0720 (0.837)	-0.5080 (0.243)	-0.5188 (0.220)	-0.0599 (0.810)	-0.0879 (0.795)	-0.0726 (0.828)	-0.4987 (0.220)	
Recur NDC			0.0094 (0.974)	0.0519 (0.931)	0.0323 (0.930)	-0.0655 (0.810)	-0.0237 (0.945)	-0.4597 (0.306)	-0.4705 (0.278)	-0.0116 (0.969)	-0.0396 (0.903)	-0.0243 (0.945)	-0.4504 (0.286)	
Recur SHC				0.0424 (0.923)	0.0228 (0.897)	-0.0749 (0.788)	-0.0332 (0.841)	-0.4692 (0.119)	-0.4800 (0.089)	-0.0210 (0.767)	-0.0490 (0.704)	-0.0338 (0.840)	-0.4598 (0.082)	
Habit NDC						-0.0196 (0.964)	-0.1174 (0.836)	-0.0756 (0.866)	-0.5116 (0.325)	-0.5224 (0.306)	-0.0634 (0.882)	-0.0914 (0.837)	-0.0762 (0.857)	-0.5022 (0.327)
Habit SHC							-0.0978 (0.784)	-0.0560 (0.794)	-0.4920 (0.153)	-0.5028 (0.126)	-0.438 (0.815)	-0.0718 (0.748)	-0.0566 (0.809)	-0.4827 (0.117)
Recur $sVRP^m$ NDC								0.0418 (0.863)	-0.3942 (0.233)	-0.4050 (0.200)	0.0539 (0.851)	0.0259 (0.923)	0.0412 (0.865)	-0.3849 (0.212)
Recur $sVRP^m$ SHC									-0.4360 (0.064)	-0.4468 (0.036)	0.0122 (0.949)	-0.0158 (0.923)	-0.0006 (0.995)	-0.4266 (0.039)
$sVRP^m + DEF + HML$										-0.0108 (0.744)	0.4482 (0.150)	0.4202 (0.110)	0.4354 (0.070)	0.0093 (0.941)
$sVRP^m + DEF + HML + R_m$											0.4590 (0.117)	0.4310 (0.076)	0.4462 (0.043)	0.0202 (0.860)
CAPM												-0.0280 (0.845)	-0.0128 (0.938)	-0.4388 (0.111)
$R_m + R_m^2$													0.0152 (0.926)	-0.4108 (0.070)
$sVRP^m$														-0.4108 (0.044)

This table presents the results of pairwise tests of equality of the OLS two-pass cross-sectional  $R^2$  values for alternative asset pricing models. We report the difference between the sample cross-sectional  $R^2$  values of the models in row  $i$  and column  $j$ ,  $\hat{R}_i^2 - \hat{R}_j^2$ , and the associated  $p$ -values in parentheses for the test of  $H_0 : \hat{R}_i^2 = \hat{R}_j^2$ . The  $p$ -values are computed under the assumption that the models are potentially misspecified.

Table 9  
Portfolio Return and Volatility Risk Premium Sensitivities to the Default Premium  
and Financial Stress, January 1996 to February 2011

<i>sVRP</i> Beta-Sorted Portfolios	Portfolio Return Default Betas with Market in the Regression	Portfolio Return Financial Stress Betas with Market in the Regression	Portfolio <i>sVRP</i> Financial Stress Betas
P1B	1.571 (2.00)	0.011 (1.61)	-0.0018 (-0.49)
P2B	1.208 (2.82)	0.012 (3.25)	-0.0021 (-1.17)
P3B	0.973 (2.15)	0.011 (2.92)	-0.0024 (-1.50)
P4B	0.521 (1.43)	0.006 (1.93)	-0.0012 (-0.85)
P5B	1.144 (3.34)	0.008 (2.66)	-0.0002 (-0.15)
P6B	0.273 (0.80)	0.003 (1.02)	-0.0005 (-0.40)
P7B	0.587 (1.85)	0.005 (1.72)	-0.0011 (-0.82)
P8B	0.612 (1.90)	0.004 (1.58)	-0.0013 (-0.99)
P9B	0.801 (2.51)	0.006 (2.29)	0.0013 (0.94)
P10B	0.077 (0.26)	0.004 (1.45)	0.0008 (0.50)
P11B	0.320 (0.98)	0.002 (0.78)	0.0003 (0.22)
P12B	-0.174 (-0.52)	-0.001 (-0.24)	0.0013 (0.76)
P13B	-0.495 (-1.57)	-0.003 (-0.92)	0.0004 (0.22)
P14B	-0.164 (-0.52)	-0.002 (-0.59)	0.0037 (1.87)
P15B	-0.046 (-0.16)	-0.001 (-0.50)	0.0039 (1.84)
P16B	-0.192 (-0.57)	-0.001 (-0.30)	0.0046 (1.96)
P17B	-0.536 (-1.60)	-0.004 (-1.31)	0.0060 (2.20)
P18B	-0.160 (-0.45)	-0.001 (-0.16)	0.0056 (1.73)
P19B	-0.720 (-2.06)	-0.006 (-2.12)	0.0098 (2.61)
P20B	-0.999 (-2.02)	-0.007 (-1.97)	0.0172 (2.91)

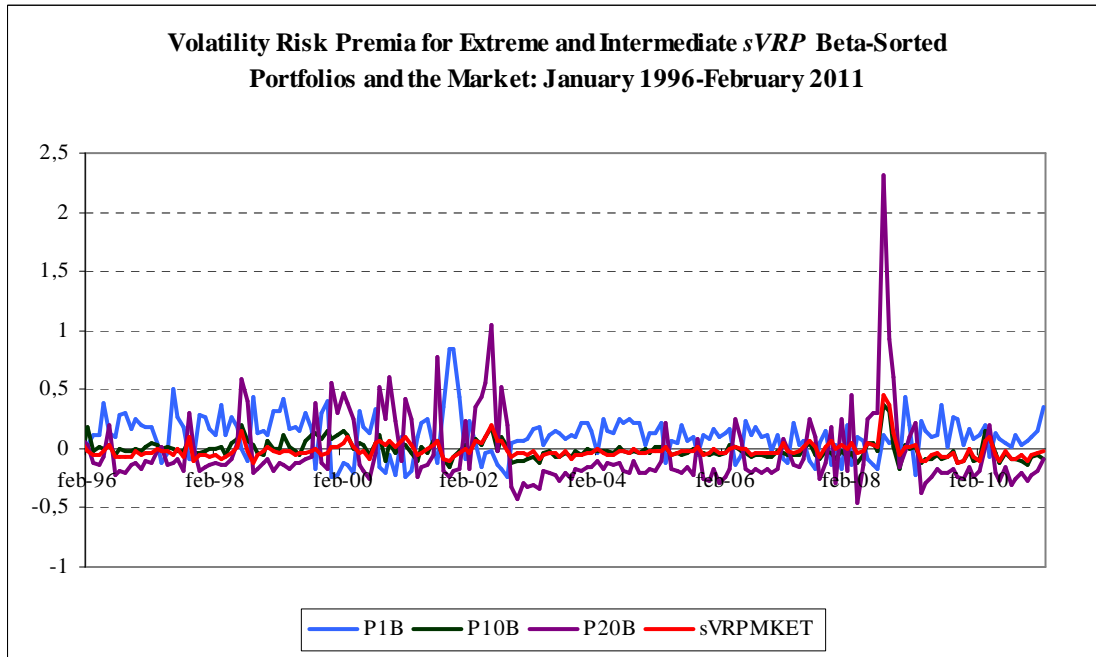
This table employs the returns of the underlying components of the 20 *sVRP* beta-sorted portfolios to estimate the default and financial stress betas controlling for market returns. The first column reports the return betas with respect to the default premium and the second column reports the betas with respect to the St. Louis Fed Financial Stress Index (STLFISI). The STLFISI measures the degree of financial stress in the markets where increasing values of the index represents higher financial stress risk. The last column displays the *sVRP* betas of the 20 *sVRP* beta-sorted portfolios with respect to STLFISI.

Table 10  
 Beta against Beta Portfolio from the Volatility Risk Premia  
 Long Low Beta and Short High Beta Portfolio Returns from the Underlying Components of the 20 Risk  
 Premia Volatility Beta-Sorted Portfolios, January 1996 to February 2011

BAB ( <i>sVRP</i> )	CAPM	Fama- French	Fama- French + MOM	Fama- French + MOM + LIQ	Excess Market Return + TED	Excess Market Return + Default
<i>Alpha</i>	0.004648 (2.454)	0.004363 (2.285)	0.004626 (2.403)	0.004643 (2.403)	0.002182 (0.868)	-0.003654 (-1.018)
<i>Adj R<sup>2</sup></i>	0.270	0.275	0.275	0.271	0.274	0.281

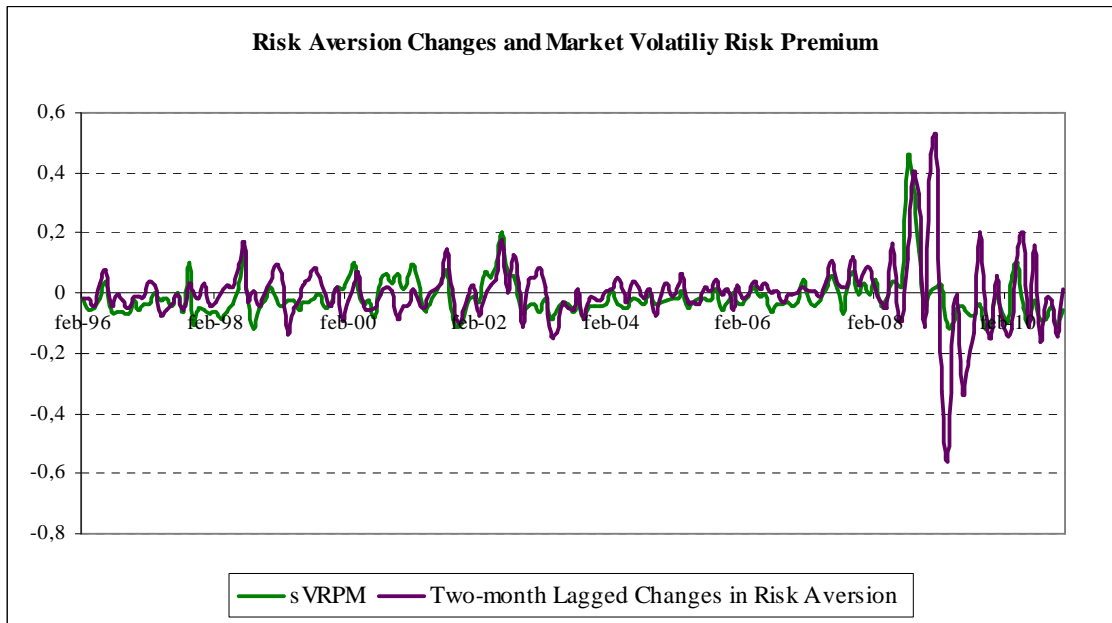
We show the results from the estimation of the OLS time-series regressions for a BAB portfolio constructed from our sample data, which is a portfolio of long levered low-beta stocks, and short de-levered high-beta securities. We report the estimated alphas for alternative factor asset pricing models. In this table, TED is a measure of funding liquidity proxied by the spread between Treasury bill rate and the euro-dollar LIBOR rate. Fama-French is the three-factor model, MOM is the momentum factor, and DEF is the default premium.

Figure 1



This figure displays the temporal behavior of the representative volatility risk premium beta-sorted portfolios and the market volatility risk premium.

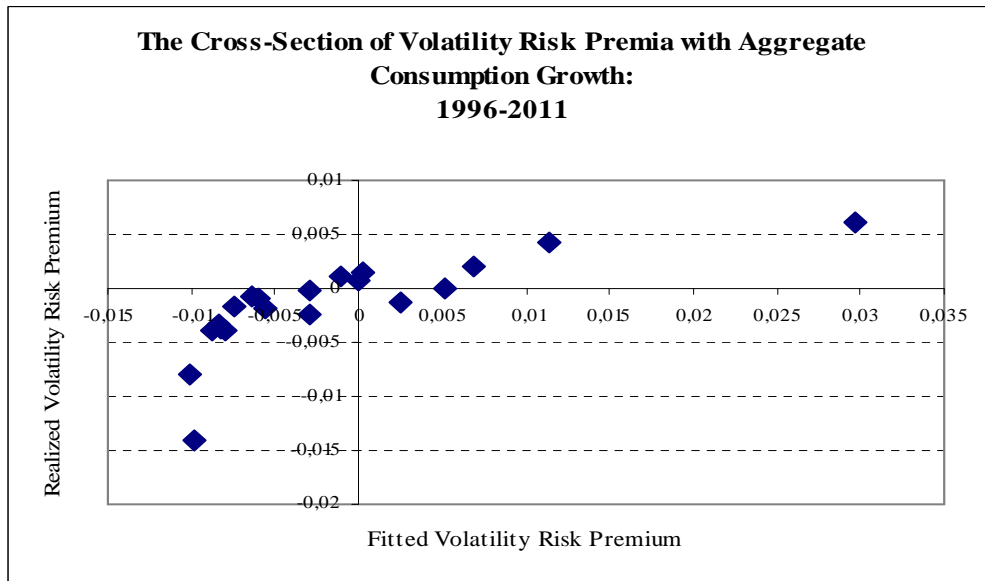
Figure 2



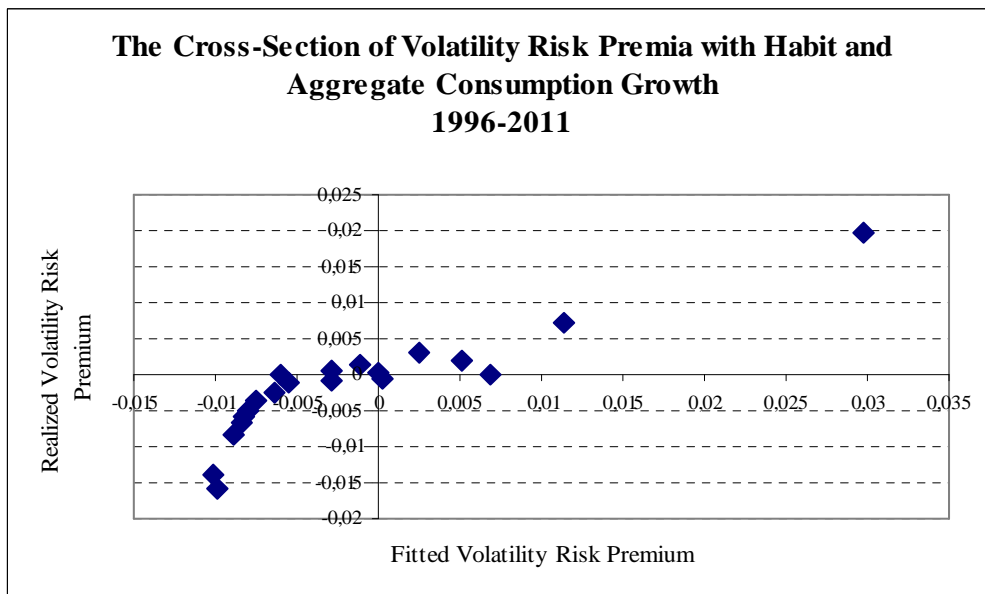
This figure displays the market volatility risk premium and time-varying risk aversion estimated under the habit preference model with a curvature parameter estimated simultaneously with the pricing model and the surplus consumption equation.

Figure 3

Average Returns versus Average Returns from the Estimated Parameters of the Fama–MacBeth Two-Pass Cross-Sectional Regression, Volatility Risk Premium Beta-Sorted Portfolios

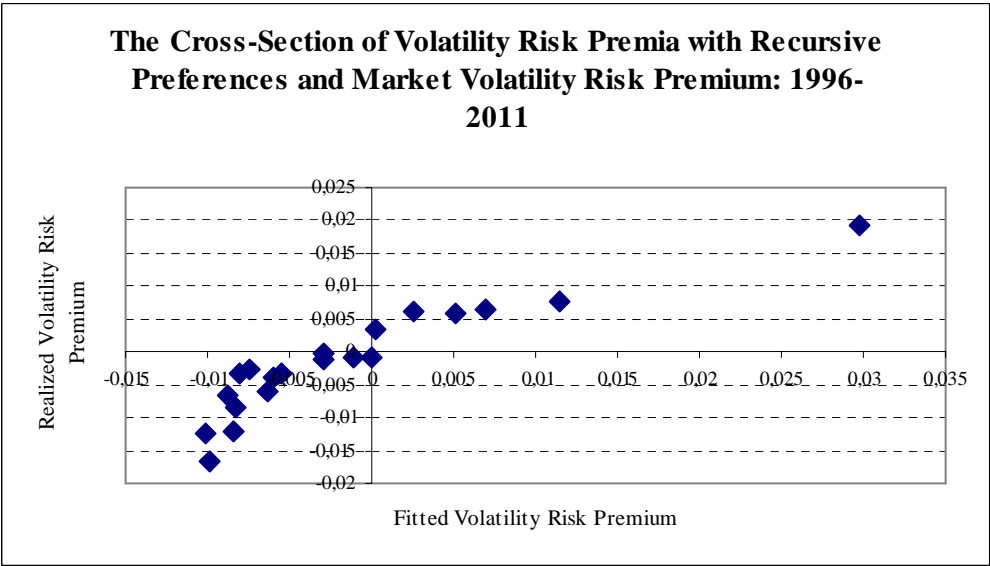


Panel A: Power utility.

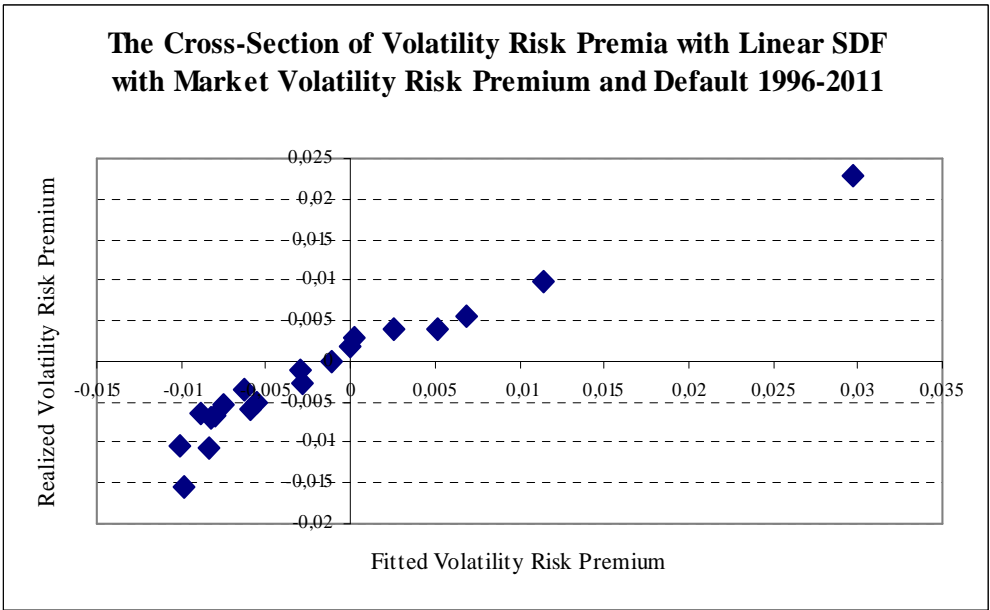


Panel B: Habit preferences with time-varying risk aversion.





Panel C: Recursive preferences with aggregate consumption growth and market wealth.



Panel D: Linear SDF with market volatility risk premium and the default premium.