Reference Prices, Double Comparisons, and Anomalies in Consumption-Payment Decisions

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Abstract

We propose a simple theoretical framework that evaluates the hedonic benefits of various consumption and payment streams. In any given period, mental accounting finds consumers engaged in a double comparison: one between the benefit of consumption and a reference price, and another between the reference price and the actual price. The reference price evolves over time, and it is influenced by the payment stream. Under standard assumptions, mainly loss aversion and adaptation, our model predicts various anomalies observed in consumer choice: sunk cost effects, the flat-rate bias, preference for advance payment, and payment depreciation. None of the existing model can explains all these biases.

1   Introduction

Commuters taking the subway could purchase a single journey ticket, a charge card, or a monthly pass. Users of a gym could sign up for a pay-as-you-go contract, a monthly contract, or a yearly contract. Drivers could make a lump sum payment and buy a car, purchase a car on a loan, or lease. Families planning their vacations could purchase a vacation home, rent, or own a time share. Prior empirical literature has shown that in situations like these consumers exhibit preferences that, on the surface, do not seem rational. For example,

1. **Flat-rate bias**: Lambrecht and Skiera (2006) and DellaVigna and Malmendier (2006) find that consumers prefer a flat-rate tariff, even though it results more expensive on average than a pay-as-you-go tariff.

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2. **Preference for upfront payment**: Prelec and Loewenstein (1998) show that consumers prefer to pay in advance, although it makes more economic sense to pay later if one considers the time value of money.

3. **Sunk-cost effect**: Thaler (1980) finds that when consumers make payment first, their later consumption decisions are affected by this investment cost, even though only the marginal cost should matter at the time of consumption.

4. **Payment depreciation**: Gourville and Soman (1998) show that the sunk-cost effect decreases over time.

In this paper, we explain these various anomalies using a utility model that evaluates the distinct hedonic benefits of various consumption and payment streams. In the single period version, the model predicts that reference prices influence willingness to pay. The model is particularly suited to evaluate repeated purchases and durable goods. Our model builds on reference effects and the psychology of (double) comparison and adaptation (Kahneman and Tversky, 1979; Thaler, 1985). Consumers keep a reference price in mind and perform two comparisons. First, the benefit of consumption are compared with the reference price, evaluating the desirability of consumption. The hedonic value of this comparison yields *acquisition utility*. Second, the reference price is compared with the actual price, evaluating the budgetary benefits of the deal. The hedonic value of this comparison yields *transaction utility*. Acquisition utility is received at the time of consumption, and transaction utility at the time of payment.

Keeping reference prices in mind is very functional, allowing consumers to initiate many purchase decisions without knowing the actual price (e.g. enter internet to book a flight or a restaurant). The purchase will be consummated if the actual price is not much higher than the reference price. For an accurate consumer, transaction utility should be an small correction due to price forecasting error, and acquisition utility should be an unbiased estimate of the rational cost and benefit of the purchase. For many consumers, however, reference prices may not be accurate forecasts, and follow instead simple adaptive rule based on the history of observed prices. This naïve adaptation rule may distort acquisition and transaction utility in a way that increases the role of reference prices in their utility.

Our model can be considered a dynamic extension of (a modified version of) Thaler (1985)'s double comparison model, one that includes reference price adaptation. Adaptation explains the flat rate bias: by using a service "for free", the naïve reference price decreases, and the acquisition utility necessarily increases. In the pay-as-you-go tariff, in contrast, the consumer is reminded of the price. Hence, the reference price, and hence the acquisition utility, is constant over time. We analyze the flat-rate bias under valuation and demand uncertainty, and the dynamic effect when consumers try to switch tariff schemes. A similar logic explains the preference for advance payment. The sunk cost effect can be purely explained using mental accounting and loss aversion, and adaptation explains payment depreciation.

Our model is primarily descriptive. For business decisions, taken on behalf of third party stakeholders, the emotional impact of reference price comparisons should not influence decisions, and one should avoid all the before-mentioned anomalies. As taught in business schools, the sunk cost effect and a preference for upfront payment are biases. A consumer that takes this business approach will be called *non-emotional*. Many consumers, however, may adopt the view that reference price effects are an integral part of the consumer experience. We call these consumers *emotional*. For emotional consumers, the model is normative and the flat rate bias, or the sunk cost effect, are rational.

A modification of discounted utility (DU) that does account for the flat-rate bias and the preference for advanced payment was introduced in Prelec and Loewenstein (1998). PL98 rests on three assumptions: prospective accounting, prorating, and coupling. Prospective accounting stipulates that utility from consumption is psychologically affected by *future* payments, and disutility from payment is psychologically affected by *future* consumption. The discrete nature of this assumption makes a consumer's utility discontinuous between prepayment and postpayment when we extend it to a continuous version. Discontinuity poses serious interpretation problems. Prorating assumes the process that one prorate the benefit of residual consumption to residual payments to buffer the pain of payment, and vice versa. Prorating is reasonable for simple decisions, but becomes psychologically implausible and mathematically intractable for large problems. Lastly, coupling refers to the degree to which consumption calls to mind thoughts of payment, and vice versa, and can be represented by coupling coefficients. Coupling coefficients are add-hoc parameters that render the

model too flexible. Our model and PL98 are subtitutes. We argue ours is more parsimonious.

Other modification of DU incorporate psychological factors such as mental accounting (Thaler, 1985, 1990, 1999), comparison with reference prices (Popescu and Wu, 2007), anticipated feelings of regret (Bell, 1982, 1985; Nasiry and Popescu, 2011), habit formation (Pollak, 1970; Wathieu, 1997; Rozen, 2010), satiation (Baucells and Sarin, 2007, 2010) or time-inconsistent preferences (DellaVigna and Malmendier, 2004). None of these models account for the type of anomalies we explain.

2 Double Comparisons in Single Purchase

Mental accounting, as proposed by Thaler (1985), postulates two sources of utility from consumption: acquisition utility and transaction utility. Acquisition utility refers to the hedonic net benefit a consumer obtains purely from consumption, as it is customary in economic models. In addition, a consumer obtains transaction utility, which refers to the hedonic benefit derived from the budgetary benefits of the deal. We adopt this single period model with a minor modification.

Let u be the benefit of consumption, i.e., the consumer's valuation of the good expressed in monetary units. Let $\hat{p}$ be a consumer's reference price and p be the actual price of the good. Also, let $v(\cdot)$ be an S-shaped value function satisfying the following properties: $v(0) = 0, v'(x) > 0$, and $v''(x) \leq 0$ for $x > 0$, and $v''(x) \geq 0$ for $x < 0$. As customary, v will exhibit loss aversion, in forms that we will define later (Tversky and Kahneman, 1991). The value function captures the emotional reaction to gains and losses. Given any such v, the consumer's utility is defined as

$$v(u - \hat{p}) + v(\hat{p} - p), \tag{1}$$

where the first term is acquisition utility and the second term is transaction utility. The consumer's valuation u admits two interpretations: a "local" willingness to pay, or a "global" consumption utility of the rational version of this consumer. In the latter case, we scale utility in such a way that the Lagrange multiplier associated with the budget constraint is one. In the normative interpretation of the model, (1) can be interpreted as the per-period experienced utility (Kahneman et al., 1997).

For future use, we define the following value function.

Definition 1. *The **power value function** is defined as follows: $v(x) = x^\gamma$ if $x \geq 0$ and $v(x) = -\lambda v(-x)$ if $x < 0$, where $0 < \gamma \leq 1$ and $\lambda \geq 1$.*

The carrier of acquisition utility is the comparison between the benefits of the purchase and the reference price. When the actual price is not available, the purchase decision will be initiated if $u > \hat{p}$, and completed if p falls below or is not too far from $\hat{p}$. The carrier of transaction utility is the comparison between the reference price and the actual price. If the actual price is equal to the reference price, a consumer perceives the transaction as a "fair deal" and she feels no additional pleasure or displeasure from the purchase and subsequent consumption. If the actual price is lower than the reference price, then she perceives that she is getting a "favorable deal" and experiences positive transaction utility. If the actual price is higher, she perceives it is being ripped-off and experiences negative transaction utility.

We define a consumer *non-emotional* iff her utility does not depend on the reference price, that is, $v(u - \hat{p}) + v(\hat{p} - p) = f(u - p)$ for some strictly increasing function f. As it turns out, the non-emotional consumer will necessarily have a linear value function and her utility simply collapses to $u - p$, the standard economic comparison of utility and cost.

Proposition 1. *The consumer is non-emotional if and only if $v(x) = cx, c > 0$.*

Reference prices are used to enter the purchased item in the mental account, and to take it out from the account at the moment of consumption. Eliminating the emotional component by setting $v(x) = x$, the accounted mental profit should be independent of the reference price. That the model admits a non-emotional consumer implies that the model respects a principle of proper accounting. When we extend the model to multiple periods, we will make sure the model has a non-emotional consumer as a particular case. The property is missing in Thaler (1985).[1]

We explore the implications of this double comparison model and the effect of reference prices on consumers' willingness-to-pay, consumption utility, and demand functions.

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[1] Thaler defines the acquisition utility as some function of u and p; and transaction utility as some function of $\hat{p}$ and p. One specification could be $v(u - p) + v(\hat{p} - p)$. This formulation is problematic, because when we rationalize the consumer with a linear value function we obtain $u - 2p + \hat{p}$, which still depends on the reference price. Moreover, because $u \geq 0$, when $\hat{p} > 2p$, a consumer would always obtain positive per-period utility from the purchase. Formulation (1) possesses two important properties: in the linear version it becomes independent of reference prices *and* does not yield the implausible conclusion that by increasing the reference price one can induce consumers to buy any item.

2.1 Willingness-to-Pay and the Beer Beach Experiment

The consumer's *willingness-to-pay* as the value of w that solves

$$v(u - \hat{p}) + v(\hat{p} - w) = 0.$$

In standard economics, the willingness to pay is equal to u, the valuation of the product. In our model, a moment's reflection reveals that $u = w$ iff $v(x) = -v(x)$. Clearly, a non-emotional consumer will satisfy this standard property. For an emotional consumer, this property is compatible with diminishing sensitivity, but not with any standard definition of loss aversion. For an emotional consumer with loss aversion and who is considering the purchase of a good, $u > \hat{p}$, the willingness to pay increases with the reference price.

Proposition 2. *Assume $u > \hat{p}$ and that loss aversion takes the form $v'(-x) > v'(x)$ for all $x > 0$. Then, w is strictly increasing in $\hat{p}$.*

In the famous beer beach experiment, Thaler (1985) asked two similar groups of people about their willingness-to-pay for the exactly same beer. One group was told that they were going to buy the beer at a fancy resort hotel and the other group was told that they were going to buy it at a small run-down grocery store.[2] He found that the median willingness-to-pay of the first group was $2.65 whereas that of the second group was $1.50, a difference he argued was difficult to reconcile with standard economic models.

This difference is natural for an emotional consumer. People expect that a fancy resort hotel would charge much more, that is, $\hat{p}_{resort} > \hat{p}_{grocery}$. If the actual price falls between these two references, the buyer will perceive the deal is favorable if buying from the resort, but feel ripped off if buying from the grocery. As predicted, the reference prices influence willingness to pay.

To illustrate, assume $u = 6$, $\hat{p}_{resort} = 4$ and $\hat{p}_{grocery} = 2$. The utility is $v(6 - 4) + v(4 - p)$ for the resort, and $v(6 - 2) + v(2 - p)$ for the grocery. Acquisition utility is positive for both, and higher for the grocery. Transaction utility is higher for the resort. The consumer will surely buy from the resort if the price is less than 4. For the grocery, however, the transaction utility becomes

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[2]The survey explained to respondents that their friends (who they trust) will buy the beer for them and the respondents will drink it on a beach, so the atmosphere of the supplier is irrelevant. Also, they were informed that bargaining is not possible. Hence, the author claims that the respondents' best strategy is to state their true reservation price for the beer.

negative if $p > 2$. Diminishing sensitivity for losses and loss aversion results in a lower willingness to pay for the grocery. Using a power value function with $\gamma = 0.9$ and $\lambda = 2.25$ yields $w_{resort} = 4.8$ and $w_{grocery} = 3.6$.

2.2 The Non-Emotional and the Emotional Consumer

Recall that the consumer will consider buying the product if $u > \hat{p}$, and will actually buy the product if $w > p$. The latter holds iff total utility is positive. For a non-emotional consumer, the purchase decision should be done iff $u > p$. An emotional consumer may incur negative utility even when $u > p$, and behave more cautiously that its non-emotional counterpart.

Proposition 3. *Assume loss aversion takes the form $-v(-x) > v(x)$, for all $x > 0$. Then,*

   i) If $u > p$ and $u \geq \hat{p} \geq p$, then $w > p$.

   ii) If $u > p$ and either $\hat{p} \leq p$ or $u \leq \hat{p}$, then w may be larger or smaller than p.

   iii) If $u = p \neq \hat{p}$, then $w < p$; and if $u = p = \hat{p}$, then $w = p$.

   iv) If $u < p$, then $w < p$.

The non-emotional and the emotional consumer exhibit the same purchasing preferences in cases i, iii-2, and iv. In cases ii and iii-1, the emotional consumer behaves more cautiously.

The beer beach example gives the intuition for the case ii-1, not buying when $\hat{p} < p \leq u$. The case case iii-1 is also intuitive. The case ii-2, a consumer that may not buy when $p < u < \hat{p}$, seems counterintuitive. Imagine an item considered to be too expensive to buy, $u < \hat{p}$, for which the consumer receives (and unsolicited) attractive offer, one with $p < u$. Because the acquisition utility is negative, she may refuse the offer (one may reason the item is ordinarily too expensive and one should not buy it). But references prices are adaptive, and repeated exposure to the bargain produces and adjustment of reference prices towards the actual price. Once $\hat{p}$ hits u, or perhaps before, the consumer will buy (one may reason that the item has gotten cheaper, and now one can afford it). By reference price adaptation, the case $p < u < \hat{p}$ will be unstable and temporary.

2.3 Demand Functions

Assume a continuum of consumers, with heterogeneous valuations for an indivisible product, say u is uniformly distributed between $[0, 1]$. In a non-emotional world, the demand function, or fraction of

consumers that will request one unit, is $d(p) = 1 - p$. What is the demand function when consumers are emotional?

Proposition 4. *Assume a continuum of consumers with valuations u uniformly distributed in $[0, 1]$. Consumers make a discrete unit consumption choice and have the power value function. The demand $d(p)$ function, or fraction of consumers for which $w > p$, is given by*

$$d(p) = \begin{cases} 1 - \hat{p} + (\hat{p} - p)/\lambda^{1/\gamma}, & \text{if } p < \hat{p}, \\ 1 - \hat{p} - (p - \hat{p})\lambda^{1/\gamma}, & \text{if } p \geq \hat{p}. \end{cases} \tag{2}$$
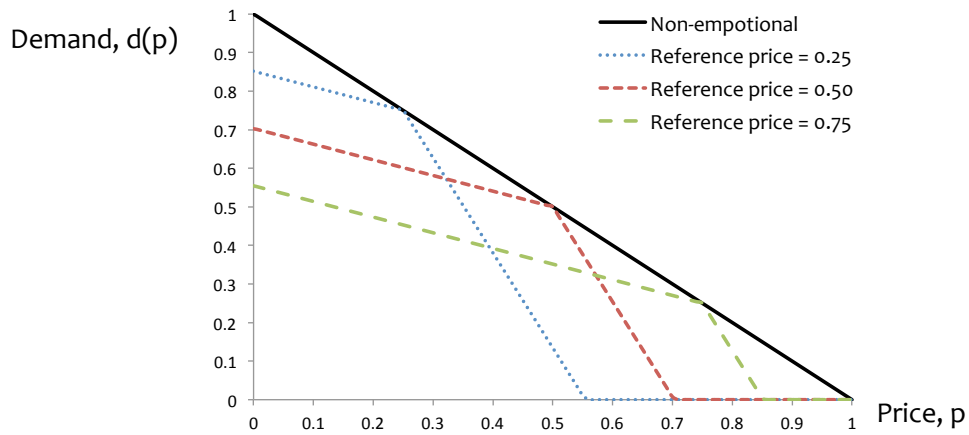

Figure 1: Demand Functions With Uniform Consumer Distribution and the Power Value Function $[\lambda = 2$ and $\gamma = 0.8]$. Non-emotional consumers exhibit $\lambda = 1$.

Figure 1 shows the demand curves for three different reference prices. Except for non-emotional consumers, the demand curve has a kink at $p = \hat{p}$. For prices above $\hat{p}$, the transaction utility is negative, and the demand drop significantly. The acquisition utility is independent of the price, and is positive for a fraction $1 - \hat{p}$ of the population. The lower the reference price, the more consumers will experience positive acquisition utility. This explains why the population of consumers with lower reference price (i.e. $\hat{p} = 0.25$) show higher demand when the price is low. In contrast, the population of consumers with a higher reference price (i.e. $\hat{p} = 0.75$) show higher demand when the price is high.

When facing a population consumers with a homogeneous reference price, $\hat{p}$, companies will tend to set prices at the current reference price. Below $\hat{p}$, the elasticity is low and the company will be tempted to increase prices. Above $\hat{p}$, the elasticity is high and the company is tempted to discount the price. This logic is correct if $\hat{p}$ is independent of p. Later, we will revisit this discussion

in the light of the adaptive nature of reference prices.

3   Double Comparisons in Repeated Purchase

We now extend the one-period double comparison model to multiple periods. The goal is to evaluate repeated purchases, or the purchase of a durable good. A period is defined as a time window during which a single episode of consumption or payment (or both) occurs. The history of consumption and payment for a certain good or service can be represented as a repetition of this time window, and based on this we can construct a consumer's consumption and payment streams. Total utility will be the sum of the per-period utility. The per-period utility will be compatible with the single purchase definition. The initial reference price is given and, from period two and on, reference prices will be determined endogenously by a process of reference-price adaptation.

In a multi-period setting, consumers may make payments for items consumed in other periods. We introduce the following vectors, all in $\mathcal{R}_+^T$: The purchase quantity vector, $\boldsymbol{\theta}$, the payment vector $\mathbf{y}$, the consumption quantity vector, $\mathbf{q}$, and the reference price vector, $\hat{\mathbf{p}}$.

$\theta_t \geq 0$ is the quantity a consumer is paying for in period t. These units may or may not be consumed during period t. For example, if $\boldsymbol{\theta} = (1, 1, ..., 1)$, then the consumer pays for one unit in each period, and if $\boldsymbol{\theta} = (T, 0, 0, ..., 0)$, then the consumer pays for T units in period 1 and nothing thereafter. If the price is constant, then $\mathbf{y} = \bar{p} \cdot \boldsymbol{\theta}$. Except for $\hat{p}_1$, the reference price is endogenously determined, and the mechanism will be described in details in section 3.1. Lastly, let $u(q_t)$ be a consumer's valuation for q_t units of consumption in one period, and has the same interpretations as u in the single-period case. u is an increasing function, with $u(0) = 0$.

The consumption and payment streams, and the payment schedule satisfy certain properties. First, in a deterministic demand setting, the total units consumed should match the total units paid for, that is $\sum_{t=1}^T q_t = \sum_{t=1}^T \theta_t$. Note that in period $t < T$, if $\sum_{i=1}^t q_i > \sum_{i=1}^t \theta_i$, then the consumer is in debt and will be making postpayment in later periods, and if $\sum_{i=1}^t q_i < \sum_{i=1}^t \theta_i$, then the consumer has pre-paid for units she may consume later. Second, if there is no purchase in period t (i.e. $\theta_t = 0$), then there is no payment (i.e. $y_t = 0$). Last, if the unit price is not the same in all periods, the average price of the units purchased in period t can be obtained as $\bar{p}_t = y_t/\theta_t$ when $\theta_t > 0$; and, importantly, $\bar{p}_t = 0$ when $\theta_t = 0$.

The per-period utility V_t in period t is defined as follows:

$$V_t = v(\delta^t u(q_t) - q_t \cdot \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - \delta'^t y_t), \tag{3}$$

where the first term is acquisition utility and the second term is transaction utility, and $0 < \delta \leq 1$ and $0 < \delta' \leq 1$ are the discount factors for valuation and payment, respectively. Observe that if $q_t = 0$, then the acquisition utility is zero; and if $\theta_t = 0$, then the transaction utility is zero.

The total utility V for the entire periods is a summation of per-period utility as follows:

$$V = \sum_{t=1}^{T} V_t. \tag{4}$$

In this dynamic context, a consumer is *non-emotional* iff her utility does not depend on the reference price, that is, $V = \sum_{t=1}^{T} f(\delta^t u(q_t) - \delta'^t y_t)$ for some strictly increasing function f. We now show that, in addition to a linear value function, a non-emotional consumer will not change her reference prices over time.

Proposition 5. *The consumer is non-emotional if and only if $v(x) = cx, c > 0$, and the reference prices do not change, $\hat{\mathbf{p}} = \mathbf{1} \cdot \hat{p}_1$.*

3.1 Adaptation

For emotional consumers, the reference price $\hat{p}_t$ may change over time when consumption and payment occur in multiple periods. The reference price may be a complicated function of past stimuli, and may also incorporate expectations consumers may form. Prices paid by oneself or peers on the same or similar products may influence the reference price. In order to provide a parsimonious model with limited degrees of freedom, we will assume the reference price is a weighted sum of past stimuli.

Given the initial reference price, $\hat{p}_1$, and the average prices in each period, $\bar{p}_\tau, \tau = 1, ..., T$, ($\bar{p}_\tau = y_\tau/\theta_\tau$ if $\theta_\tau > 0$, and $\bar{p}_\tau = 0$ if $\theta_\tau = 0$), the reference prices are given by

$$\hat{p}_{t+1} = \alpha_{t,0}\hat{p}_1 + \sum_{\tau=1}^{t} \alpha_{t,\tau}\bar{p}_\tau, \quad t = 1, ..., T-1, \tag{5}$$

where $\alpha_{t,\tau} \geq 0, t = 1, ..., T-1; \tau = 1, ..., t$ is the effect of the price in τ on the reference price in period $t + 1$; and $\alpha_{t,0} \geq 0, t = 1, ..., T-1$ is the effect of the initial reference price on the reference

price in period $t + 1$. We assume $0 < \sum_{\tau=0}^{t} \alpha_{t,\tau} \leq 1$.

To obtain insightful results, we assume $\sum_{\tau=0}^{t} \alpha_{t,\tau} = 1$. This constraint ensures that if $\hat{p}_1 = p_t = p$, then $\hat{p}_t = p$. Weight will exhibit *total adaptation* if $p_t \to p$ implies $\hat{p}_t \to p$. Weights will exhibit *minimal adaptation* if $\alpha_{t,t} \geq \alpha$, for some $\alpha > 0$.

We focus on two specifications, characterized by one parameter, the speed of adaptation $\alpha = \alpha_{t,t}$.

RA. **The recency-weighted average**: For $0 < \alpha < 1$, let

$$\hat{p}_{t+1} = (1 - \alpha)\hat{p}_t + \alpha\bar{p}_t, \quad t = 1, ..., T - 1.$$

AFL. **The average of the first and the last price**: For $0 < \alpha < 1$, let

$$\hat{p}_{t+1} = (1 - \alpha)\hat{p}_1 + \alpha\bar{p}_t \quad t = 1, ..., T - 1.$$

RA is widely used in marketing modeling (Mazumdar et al., 2005). RA can be written as a recursion, and exhibits total adaptation. The implicit weights are $\alpha_{t,0} = (1 - \alpha)^t$, and $\alpha_{t,\tau} = \alpha(1 - \alpha)^{t-\tau}, 1 \leq \tau \leq t$. RA exhibits total adaptation if $\alpha > 0$.

AFL is simple, and has some empirical support. Baucells et al. (2011) present a sequence of experiments showing that, in the formation of reference prices, the intermediate prices received a small weight compared to the first and last price. AFL captures this effect of primacy and recency, often found in psychology. The implicit weights are $\alpha_{t,0} = (1 - \alpha)$, $\alpha_{t,t} = \alpha$, and $\alpha_{t,\tau} = 0$ for $1 \leq \tau \leq t - 1$, which exhibit minimal adaptation if $\alpha > 0$.

Our model is parsimonious, in the sense that the value function and the reference price rule is "universal" to any reference-dependent model and can be externally validated. Given the wide acceptance of reference-dependence in psycho-economic models, it is an inescapable task of social sciences to estimate how reference points are determined and evolve over time. Our view is that reference updating rules are simple hard wired rules. These naïve learning rules can be overruled by explicit thinking (Kahneman, 2011). It is an empirical question to determine the particular form of the updating rule, although some evolutionary models may provide the foundation.[3]

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[3]Parsimony is lacking in PL98. The coupling coefficients of are proper to the model. Moreover, PL98 fails to distinguish many different streams of consumption and payments. For example, their model would predict that a

Henceforth, the default assumption is that prices are constant, $\mathbf{y} = \bar{p} \cdot \boldsymbol{\theta}$, and that there is no discounting, $V_t = v(u(q_t) - q_t \cdot \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - y_t)$.

4   Preference for Advance Payment

Prelec and Loewenstein (1998) argue that people generally like to pay first and consume later, a preference that is at odds with the most elementary notions of finance and the time value of money. For example, 60 percent of the people expressed a preferred to prepay for a one-week vacation to the Caribbean. To examine the difference between prepayment and postpayment in our model, we first define the payment schedules $\boldsymbol{\theta_n^a}$ and $\boldsymbol{\theta_n^b}$ as follows.

Definition 2. $\boldsymbol{\theta_n^a}$ *is defined as* $\theta_i = n$ *if i is a multiple of n, and $\theta_i = 0$ otherwise.*

Definition 3. $\boldsymbol{\theta_n^b}$ *is defined as* $\theta_i = n$ *if $(i + n - 1)$ is a multiple of n, and $\theta_i = 0$ otherwise.*

Under $\boldsymbol{\theta_n^a}$, consumers make postpayment for n units every nth period; and under $\boldsymbol{\theta_n^b}$, consumers prepay for n units every nth period. For example, $\theta_3^a = (0, 0, 3, 0, 0, 3, \ldots, 0, 0, 3)$ and $\theta_3^b = (3, 0, 0, 3, 0, 0, \ldots, 3, 0, 0)$. Consider a constant consumption stream for a known number of periods, $T \geq 2$, with constant prices and no discounting. For a simple comparison, we set $n = T$ and compare $\boldsymbol{\theta_n^a} = (0, 0, ..., T)$ with $\boldsymbol{\theta_n^b} = (T, 0, 0, ..., 0)$.

Proposition 6. *Assume that loss aversion takes the following form:* $-v(-x) \geq v(x)$ *for all $x > 0$. Let $\mathbf{y} = p \cdot \boldsymbol{\theta}$ and $\mathbf{q} = \mathbf{1}$. Under RA and AFL, consumers always prefer prepayment to postpayment, i.e.,* $V(\boldsymbol{\theta_T^b}) > V(\boldsymbol{\theta_T^a})$.

Shafir and Thaler (2006) showed that when people buy wines in advance, they think of it as an "investment," and later when they consume the wines, they feel as if the wines were "free." Reference price adaptation, and the assumption that $\bar{p}_t = 0$ if $\theta = 0$, implies that the acquisition utility becomes larger and larger as one gets used to not paying. Further evidence for this effect is provided by Gourville and Soman (1998), who showed that when a consumer makes payment in advance, her attention to this payment will gradually decrease over time. The flip side is a decrease in future transaction utility if one re-purchases a service at a cost that now seems expensive.

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prepayment of one hour and a prepayment of one year would provide the same hedonic benefit to consumers. In our case, because of adaptation, there would be a difference between these two.

Period	1	2	3	4	5	6	7	8	9	10	Total
$u(q_t)$	1	1	1	1	1	1	1	1	1	1	12.00
θ_t	4	0	0	0	4	0	0	0	4	0	12.00
y_t	2	0	0	0	2	0	0	0	2	0	6.00
RA Reference Price	0.50	0.50	0.25	0.13	0.06	0.28	0.14	0.07	0.04	0.07	
Acquisition Utility	0.57	0.57	0.79	0.90	0.95	0.77	0.89	0.94	0.97	0.78	9.98
Transaction Utility	0.00	0.00	0.00	0.00	-3.13	0.00	0.00	0.00	-3.28	0.00	-6.41
Per-period Utility	0.57	0.57	0.79	0.90	-2.18	0.77	0.89	0.94	-2.31	0.78	3.56

Table 1: Comparison of Pre-payment with Post-payment. Consumption of $\mathbf{q} = 1$. [$\bar{p} = 0.5, u(1) = 1, \alpha = 0.5, \gamma = 0.8,$ and $\lambda = 2$)

Specifically, this is how our model captures the result from Shafir and Thaler (2006). Assume a consumer has bought 10 bottles of wine at the price of \$20 for each bottle, and will consume each bottle over 10 separate occasions (or periods). First, if the reference price does not change, then $\hat{p}_t = \hat{p} = \$20$ for all t. From an accounting viewpoint, the purchase of 10 bottles in period 1 is considered an investment having a book value of $10\hat{p} = \$200$. We pay $y_1 = \$200$ for such an investment, producing a capital gain of $10\hat{p} - \$200 = \$200 - \$200 = \$0$, whose psychological value is $v(10\hat{p} - \$200) = v(\$0)$, which is transaction utility. We amortize the bottles as we consume. In period 2, we consume one bottle, and realize a benefit of $u(1) = u$ (the revenue of selling the bottle to our stomach and getting utility in return) minus the book cost of $\hat{p} = \$20$. The total benefit is $u - \hat{p} = u - \$20$, and its psychological value is perceived as $v(u - \$20)$, which is acquisition utility. If $v(\cdot)$ is linear, then the gain per bottle is u minus the average cost; and the total gain is $10u - \$200$, as one would expect. On the other hand, if the reference price changes, say $\hat{p}_2 < \hat{p}_1 = \$20$, then there is a capital loss of $\hat{p}_1 - \hat{p}_2 = \$20 - \hat{p}_2$ that we fail to acknowledge. This produces a larger perceived benefit at the moment of consumption, as if the cost of such bottle were now cheaper. It produces, however, a larger capital loss at the moment of re-purchasing a set of bottles of the same price, as these will be perceived as more expensive.

5   Sunk Cost Effects

Thaler (1980) suggested that once people have made an up-front payment for future consumption, their consumption decision is affected by this sunk cost, even though traditional economic theory stipulates that only the marginal cost at the time of consumption should affect the decision. This is called the "sunk-cost effect."

Losing or giving up a pre-purchased item is painful. In the model, if some pre-purchase

quantities are lost or become obsolete, then there is a mental amortization. Suppose we have pre-purchased a consumption quantity vector, $\mathbf{q}$, and $\tau \geq 1$ the period where the items become obsolete. In this case, the term $\sum_{t=\tau}^{T} v(u - \hat{p}_t)$ in V gets replaced by the mental book loss of $v(-(T-\tau+1)\hat{p}_\tau)$. In other words, mental amortization of a pre-purchased quantity stream of $\mathbf{q}$ at τ is defined as:

$$V^\ell = V - \sum_{t=\tau}^{T} v(u - \hat{p}_t) + v(-(T - \tau + 1)\hat{p}_\tau).$$

Suppose you bought an smart phone, anticipating a per-period benefit of u during T periods. The decision was made because the lifetime benefit exceeds the price, $uT > p$. Suppose that after having paid for the phone, and before using it, you learn that a new model costing also p gives you $2u$. The old model cannot be sold because is obsolete. Would you buy the new model? A non-emotional consumer will always replace the old model with the new one because $2uT - p - p > uT - p$ necessarily holds if $uT > p$ in the first place.

How would an emotional consumer react? Assume $p \leq T\hat{p}_1 < Tu$, which ensures the emotional consumer buys the old phone to begin with. Buying and keeping the old model produces:

$$v(T\hat{p}_1 - p) + \sum_{t=1}^{T} v(u - \hat{p}_t) > 0.$$

Buying the old model and switching to a new one at $\tau = 1$ produces:

$$v(T\hat{p}_1 - p) + v(-T\hat{p}_1) + v(T\hat{p}_1 - p) + \sum_{t=1}^{T} v(2u - \hat{p}_t) = v(-T\hat{p}_1) + Tv(2u - \hat{p}_1).$$

If v is linear and the reference price is constant, then switching to the new model, $2uT - 2p$, is always better than keeping the old one, $uT - p$.

For an emotional consumer, the mental amortization of the first model, $v(-T\hat{p}_1)$, is painful. To see that loss aversion alone explains the effect, assume that the price is fair, $p = T\hat{p}_1$, the reference prices stay constant, $\alpha = 0$, and the value function is piecewise linear, $\gamma = 1$. To generalize, assume the new model reports benefits of $\theta u, \theta \geq 2$. The emotional consumer will *not* switch to the new model iff $\theta uT - p - \lambda p < uT - p$, or

$$\lambda > (\theta - 1)\frac{uT}{p}.$$

For an emotional consumer, the cost/benefit ratio of the new model needs to be $\lambda + 1$ times as good

the original one to justify the switch.

Gourville and Soman (1998) showed that the intensity of the sunk cost effect decreases with the passage of time. They analyzed the attendance records of a gym, where all customers had one-year memberships and made payments twice a year. They found out that the attendance was the highest in the month when payment was made, and it has steadily decreased over time afterwards.

5.1 The Unused Unit Effect

Suppose that a consumer has bought a season ticket for basketball games thinking that he would go to 10 games, but on one of the game days an important business meeting came up and he could not make it to the game. Then, naturally he would obtain negative utility from not being able to go to the game he already paid for.

In the reference-dependent utility model, under a multi-period setting, we assumed that $\sum_{t=1}^{T} q_t = \sum_{t=1}^{T} \theta_t$ with deterministic demand. However, if a consumer forgoes consumption in a certain period unexpectedly, then $\sum_{t=1}^{T} q_t < \sum_{t=1}^{T} \theta_t$, and therefore we need a minor modification of the per-period utility in a multi-period setting. Suppose that the total period is T, $\sum_{t=1}^{T} \theta_t = T$ and $\sum_{t=1}^{T} q_T < T$. Then, since a consumer is paying for consumption in every period, the consumer's per-period utility should be defined as follows:

$$V_t = v(u(q_t) - \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - y_t), \tag{6}$$

where the reference price $\hat{p}_t$ enters in acquisition utility regardless of q_t. Therefore, in the basketball game example, when the consumer cannot make it to the game, he would incur negative acquisition utility of $v(-\hat{p}_t)$ where $\hat{p}_t$ is the reference price at that time. Note that right after the consumer has made payment, under a flat-rate tariff, $\hat{p}_t$ would be so high that he would feel high mental pain of forgoing consumption, but the reference price decays over time and the disutility of forgoing consumption also decreases over time as well.

Proposition 7. *Under a flat-rate tariff $\boldsymbol{\theta_T}$ with deterministic demand, consumers experience negative per-period utility $v(-\hat{p}_t)$ when they forgo consumption in period t. This disutility of forgoing consumption strictly decreases over time when the reference price is updated by RA, and it decreases in the third period and persists over time when the reference price is updated by AFL.*

Proposition 7 makes intuitive sense. In the beginning of a flat-rate contract, the large up-front payment saliently remains on consumers' mind and consumers feel significant mental pain when they have to forgo consumption for some external reasons. However, if the reference price strictly decreases over time (e.g. by RA), people gradually adapt to using the service for free and they do not feel as much pain from not using the service as they did in the beginning. For example, when people sign up for a gym with a one year contract, they may be eager to go to the gym as frequently as possible in the beginning. Since the big payment they made when they signed up still remains saliently in their mind, it feels painful when they cannot go to the gym. But as time passes, the salience of the initial payment gradually dissipates (i.e., the reference price decreases), and it feels less painful not to go to the gym. This may be one explanation why people prefer to choose a flat-rate contract when signing up for a gym, but later they do not go to the gym that much, so they actually could have saved money under a pay-as-you-go contract (DellaVigna and Malmendier, 2006; Gourville and Soman, 1998).

6 Tariff Choices in Repeated Consumption

Consumers have a tendency to prefer flat-rate tariffs even when they could save money under pay-per-use tariffs. DellaVigna and Malmendier (2006) studied the contractual choices of consumers at gyms and found that 80 percent of consumers under the flat-rate contract would have saved money under the pay-per-visit contract. The extra-cost was as large as 70 percent for those consumers who chose a membership of over \$70 per month. Lambrecht and Skiera (2006) studied the tariff choices of Internet access and found that a significant fraction of consumers consistently chose a tariff with a higher fixed-fee with a higher allowance over multiple periods, even though they could have saved money under a tariff with a lower fixed-fee with a lower allowance.[4] More than half of the consumers with the flat-rate bias paid at least 100% more than they would have on the least costly pay-per-use tariff.

We first identify the flat-rate bias with deterministic demand and valuation. As before, there is a firm providing a service to consumers for T periods and a consumer makes a discrete consumption

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[4]Lambrecht and Skiera (2006) analyzed the transactional data of 10,882 customers of an Internet service provider. Their Table 1 shows that, over 5 months, 46.4% of consumers under Tariff 2 (which had a higher fixed-fee with a higher allowance) would have been on average better off under Tariff 1 (which had a lower fixed-fee with lower allowance). They compare three different tariffs over 3 month and 5 month periods with two different criteria.

choice in each period. We consider a consumer that purchases n units at a time. In the case of $n = 1$, this corresponds to a pay-as-you-go scheme. In the case of $n = T$, and certainty about T, this corresponds to a flat-rate fee. The case of $1 < n < T$ corresponds to a pre-purchase card of n uses (see Table 2). Formally, we will explore preferences over purchase quantity vectors, $\boldsymbol{\theta}_n^b$. Recall that, for example, $\boldsymbol{\theta}_2^b = (2, 0, 2, 0, ...)$ and that $\boldsymbol{\theta}_3^b = (3, 0, 0, 3, 0, 0, ...)$.

Throughout, we assume unit consumption, $\mathbf{q} = \mathbf{1}$, constant price, $\mathbf{y} = p\boldsymbol{\theta}$, and no discounting. For ease of comparison, we set the initial reference price at the average price, $\hat{p}_1 = p$, and normalize $u(1) = 1$. We consider the case when the consumer knows with certainty the number of uses, T, and the case of uncertainty on how often she will use the service because the per-use valuation is uncertain, or the number of periods is uncertain.

	Period	1	2	3	4	5	6	7	8	9	10	11	12	Total
	$u(q_t)$	1	1	1	1	1	1	1	1	1	1	1	1	12.00
	θ_t	4	0	0	0	4	0	0	0	4	0	0	0	12.00
	y_t	2	0	0	0	2	0	0	0	2	0	0	0	6.00
AFL	Reference Price	0.50	0.50	0.25	0.25	0.25	0.50	0.25	0.25	0.25	0.50	0.25	0.25	
	Acquisition Utility	0.57	0.57	0.79	0.79	0.79	0.57	0.79	0.79	0.79	0.57	0.79	0.79	8.65
	Transaction Utility	0.00	0.00	0.00	0.00	-2.00	0.00	0.00	0.00	-2.00	0.00	0.00	0.00	-4.00
	Per-period Utility	0.57	0.57	0.79	0.79	-1.21	0.57	0.79	0.79	-1.21	0.57	0.79	0.79	4.65

Table 2: Hedonic Value of a Repeated Purchase of a Card of Valid for $n = 4$ uses. [$\mathbf{q} = \mathbf{1}$, $\bar{p} = 0.5, u(1) = 1, \alpha = 0.5, \gamma = 0.8,$ and $\lambda = 2$)

6.1 Flat-rate Bias under Certainty

First, we compare a consumer's utility under a pay-as-you-go tariff and a flat-rate tariff. Under a pay-as-you-go tariff, a consumer makes payment in every period. If $\sum_{\tau=0}^{t} \alpha_{t,\tau} = 1$, then the reference price $\hat{p}_t$ remains at the unit price p throughout the periods, and the transaction utility is zero. Because $u(1) = 1$ and $\hat{p}_1 = p$, the hedonic value of a simple pay-as-you-go scheme is

$$V(\boldsymbol{\theta}_1^b) = \sum_{t=1}^{T} v(1 - \hat{p}_t) = n \cdot v(1 - p). \tag{7}$$

Under a flat-rate tariff, a consumer pays everything in the first period and nothing thereafter, so the reference price $\hat{p}_t$ decays throughout the periods (recall that if $\theta_i = 0$, then $\bar{p}_i = 0$). Transaction utility occurs only in periods of payment, that is, in the first period. Because $\hat{p}_1 = p$, we have that

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$\theta_1 \hat{p}_1 = Tp = y_1$. Therefore, transaction utility is zero and

$$V(\boldsymbol{\theta_T^b}) = \sum_{t=1}^{T} v(1 - \hat{p}_t). \tag{8}$$

For a consumer with no changes in reference prices, such as the non-emotional type, $V(\boldsymbol{\theta_1^b}) = V(\boldsymbol{\theta_T^b}) = nv(1 - p)$. If reference prices are minimally adaptive, however, a flat-rate tariff will be strictly preferred to a pay-as-you-go tariff.

Proposition 8. *If $T > 1$ and $\alpha_{t,t} > 0$, then $V(\boldsymbol{\theta_T^b}) > V(\boldsymbol{\theta_1^b})$.*

PL98 explain the flat-rate bias using a complicated mechanism of coupling and prospective accounting. Our explanation is based on a natural mechanism of double comparison and adaptation. The use of adaptation, a general psychological principle, to explain the flat rate bias is novel.

In our numerical exploration of pre-purchase cards, we find that $V(\boldsymbol{\theta_n^b})$ increases with n under a broad set of parameter values and adaptation rules, provided n is a divisors of T. The result is violated under RA and n is small (see Table 3). Under AFL, the more uses a pre-purchase card has, the more hedonic value it has. Consequently, flat-rate is preferred to a pre-pruchase card, and a pre-pruchase card is preferred to pay-per-use. Formally,

Proposition 9. *Under AFL,*

$$V(\boldsymbol{\theta_n^b}) = \left(1 + \frac{T}{n}\right)v(1-p) + \left(T - 1 - \frac{T}{n}\right)v(1 - (1-\alpha)p) + \left(\frac{T}{n} - 1\right)v(-\alpha np).$$

Let n and n' be divisors of T. If $\alpha > 0$ and $1 \le n < n' \le T$, then $V(\boldsymbol{\theta_{n'}^b}) > V(\boldsymbol{\theta_n^b})$.

	n	1	2	4	5	8	10	16	20	40	80
RA	$\gamma=0.8$	45.9	6.9	8.6	11.9	20.7	25.2	34.9	39.7	56.4	78.7
	$\gamma=0.5$	56.6	5.4	22.4	28.6	40.9	46.0	55.5	59.4	69.9	79.2
AFL	$\gamma=0.8$	45.9	9.7	20.9	23.9	29.8	32.4	38.0	40.7	50.3	63.1
	$\gamma=0.5$	56.6	7.6	27.9	33.0	42.1	45.7	52.3	55.1	62.5	69.0

Table 3: Hedonic value of pre-purchase cards, $V(\boldsymbol{\theta_n^b})$, as a function of n and the adaptation process. $[T = 80, u(1) = 1, \alpha = 0.5,$ and $\lambda = 2]$

6.2 Tariff Choice under Demand Uncertainty

Existing literature have shown consumers to have a biased preference for flat-rate tariffs even if they face demand uncertainty (Lambrecht and Skiera, 2006; Narayanan et al., 2007; DellaVigna

and Malmendier, 2006; Grubb, 2009; Miravete, 2002, 2003; Goettler and Clay, 2009). We will show that reference price adaptation successfully explains this finding.

At the moment of signing a flat rate contract, or buying a pre-purchase card, consumers often do not now the exact use they will make of the service. Two sources of uncertainty seem relevant. One is the valuation of the service as time progresses. For example, the value of going to the gym may vary over time because of unforeseeable time availability constraints, or tiredness and satiation effects. The second effect, which we consider in the next subsection, is the risk of product obsolescence.

We first examine the effect of uncertainty in the valuation. Let T be fixed and known, and assume the consumers' valuation, $u_t(q_t)$, is uncertain. The consumer will hold some (joint) probability distribution about the realization of $u(q_t)$, and the actual realization will be known at t, before deciding whether to use the service or not. We restrict ourselves to independent realizations, the value of $u(q_t)$ has not forecasting value, and the consumer will choose q_t so as to maximize the per-period utility.

For a simple comparison, we compare a flat-rate tariff and a pay-as-you-go tariff. We consider a discrete choice problem, let $\hat{p}_1 = p$ and assume $\sum_{\tau=0}^{t} \alpha_{t,\tau} = 1$. Under a flat-rate tariff, the payment schedule is $\boldsymbol{\theta_T^b}$ as before. The per-period utility of using the service is always higher than that of not using the service because of the up-front payment. Consumption, however, may not bring positive feelings if $u_t < \hat{p}_t$. The acquisition utility is negative because one is using a service whose assigned cost is higher. One feels such consumption is not justified, and would not have been done under a pay-as-you-go scheme. Under AFL, these feelings will stay over time as long as $u < (1-\alpha)p$. In the RA model, these feelings will disappear over time, as $\hat{p} \to 0$, and one will act as if the service were truly for free. In any case, the consumer will always use the service because $v(u_t - \hat{p}_t) > v(-\hat{p}_t)$ and $\mathbf{q} = \mathbf{1}$.

Under a pay-as-you-go tariff, a consumer would skip the service if the willingness to pay is below the price. Note that if we skip the purchase, there is no acquisition and no transaction, and by our definition of period, there is no reference price adaptation. As in the case of certainty, pay-as-you-go leads to $\hat{p}_t = p, t = 1, ..., T$ and transaction utility is zero. Therefore, $w_t = u_t$, and the payment schedule is $\tilde{\boldsymbol{\theta}}_1^b$, where $\tilde{\theta}_t(u_t) = 1$ if $u_t \geq p$ and $\tilde{\theta}_t(u_t) = 0$ if $u_t < p$.

Let $V(\boldsymbol{\theta_T^b})$ and $V(\tilde{\boldsymbol{\theta}}_1^b)$ denote the total utility under a flat-rate and a pay-as-you-go tariff, respectively. The expected total utility is given by:

$$E[V(\boldsymbol{\theta_T^b})] = E_u[\sum_{t=1}^{T} v(u - \hat{p}_t)],$$

$$E[V(\tilde{\boldsymbol{\theta}}_1^b)] = E_u[\sum_{t=1}^{T} \max\{v(u - p), 0\}].$$

In the pay as you go case, $E[V(\tilde{\boldsymbol{\theta}}_1^b)]$ involves a maximum operator, because consumers can skip consumption whenever $v(u_t - p) < 0$.

It would seem that, under uncertainty, the pay-as-you-go tariff is superior, as it gives more flexibility. The flat rate bias, however, retains the hedonic value of a higher acquisition utility due to reference-price adaptation. Moreover, once the consumer gets adapted to using the service for free, she will use the service more often, further increasing the experienced utility. If reference prices were not to change, the buyer of a flat-rate contract would feel negative acquisition utility every time it uses the service when its valuation is low, $u_t < \hat{p}$. Because of reference price adaptation, these negative feeling vanish over time. Hence, if these negative feelings are under control because u does not fall too much below p, or T is sufficiently large, the flat-rate tariff will be preferred over the pay-as-you-go tariff.

Without loss of generality, assume u is continuously distributed on $[0, 1]$ (any other distribution can be approximated by such family). Because $\hat{p}_1 = p$, there is no transaction utility in the flat-rate and, as argued, pay-as-you-go does not have transaction utility either. Moreover, $\hat{p}_1 = \hat{p}_2 = p$ and, for $t \geq 3$, $\hat{p}_t \leq (1 - \alpha)p < p$. The difference between the two contracts, $E[V(\boldsymbol{\theta_T^b})] - E[V(\tilde{\boldsymbol{\theta}}_1^b)]$, is given by

$$\sum_{t=1}^{T} \int_p^1 f(u)[v(u - \hat{p}_t) - v(u - p)]du + \sum_{t=3}^{T} \int_{\hat{p}_t}^p f(u)v(u - \hat{p}_t)du + \sum_{t=1}^{T} \int_0^{\hat{p}_t} f(u)v(u - \hat{p}_t)du. \quad (9)$$

The first and second terms are positive, strictly so if $T \geq 3$ and $\alpha > 0$. In general, the utility of the flat rate is higher, except for periods in which the valuation if below the reference price. If valuations satisfy $P(u_t < \hat{p}_t) = 0$ and $T \geq 2$, then we have that flat-rate is strictly preferred to pay-as-you-go. This requires $u_1, u_2 > p$, a condition that may fail. A more reasonable approach is to assume that T is sufficiently large, so that we compensate for this negative expected utility. We

now show sufficient conditions that ensure that the flat-rate preference generalized to the case of uncertain valuation.

Proposition 10. *Consider a discrete choice repeated purchase with uncertain valuation, with u_t iid and $E[u_t] > p$. The consumer prefers a flat-rate tariff over a pay-as-you-go tariff if reference prices are minimally adaptive, T is sufficiently large and*

$$\sum_{t=1}^{\infty} v(-\hat{p}_t)P(u_t \leq \hat{p}_t) > -\infty. \tag{10}$$

It is not difficult to find sufficient conditions that ensure (10) is bounded:

1. Reference prices are minimally adaptive and $P(u < (1-\alpha)p) = 0$.

2. Reference prices are totally adaptive and for some $\epsilon > 0$, $P(u < \epsilon) = 0$.

3. Reference prices follow RA, $\alpha > 0$, v is power, and u_t is any idd process.

The result stresses again the crucial role that adaptation processes have on the flat-rate bias. In the case of uncertain valuation, however, T has to be large to justify this preference. Not surprisingly, flat-rate tariffs are often offered for relatively large time periods (two years for many phone contracts, or lifetime club memberships or lifetime ownership of time-share contracts). For durable goods, and uncertainty over the valuation of the good in any given period, the preference for ownership over rental will increase with the duration of the good.

The result holds for important examples such as u_t being a Bernoulli random variable with $P(u = 1) > \pi$ and $P(u = 0) = 1 - \pi > 0$. For example, the value of u_t could be determined by the time availability to use the service. If one is able to use the service, then the valuation is always $u_t = 1$.

6.3 Tariff Choice under Risk of Obsolescence

A tourist in a city usually faces the choice of buying a single-use ticket or a weekly/monthly pass for transportation, but they are rarely certain about how much they will use transportation. Also, when signing up for a gym, consumers do not know how often and for how long they will end up using the facilities. Consumers quite often face the decision of having to choose a tariff not knowing for how long they need the service.

Again, we compare a flat-rate tariff and a pay-as-you-go tariff. Rather than assuming T is random, let $T = \infty$ and let the demand be given by $\mathbf{q_x} = (1, 1, \ldots, 1, 0, 0, 0, \ldots)$, where the first x components are ones, and the rest are zeros.

Consumers choose a tariff between the two options based on the demand distribution prior to consumption. After choosing a tariff, consumers start using the service from period 1 and on, until the product becomes obsolete at $t = \tau$. Let $n = E[\tau]$. and at the period when they decide to stop using the service, they realize the actual demand, which is the total number of periods they have used the service. Under a flat-rate tariff $\boldsymbol{\theta_n^f}$, consumers pay a fixed amount np up-front, and can consume the item as many times as possible. Naturally, if a consumer's realized demand is less than n, she would incur negative utility for having paid more than what she would actually consume. In the same manner, if a consumer's realized demand is larger than n, she would obtain positive utility from period $n + 1$ for being able to consume more than what she has paid for. Considering these two cases, we can obtain the total utility under a flat-rate tariff, $V(\boldsymbol{\theta_n^f}, \mathbf{q_x})$, as a function of payment stream $\boldsymbol{\theta_n^f}$ and consumption stream $\mathbf{q_x}$ as follows:

$$V(\boldsymbol{\theta_n^f}, \mathbf{q_x}) = \begin{cases} \sum_{i=1}^{x} v(1 - \hat{p}_i) + v(-(n-x)\hat{p}_x), & \text{if } x \leq n, \\ \sum_{i=1}^{n} v(1 - \hat{p}_i) + (x - n)v(1), & \text{if } x > n. \end{cases} \tag{11}$$

Note that transaction utility does not enter, because consumers make payment only in the first period when the reference price is equal to the actual price. When $x < n$, a consumer incurs disutility of $v(-(n-x)\hat{p}_x)$ in period x, because she realizes that she paid for $(n-x)$ more units than she actually consumed. Also, when $x > n$, she obtains acquisition utility of $v(1)$ in each period from period $n + 1$, because the consumption is "free" and therefore the reference price does not come into her acquisition utility. Hence, she obtains additional utility of $(x - n)v(1)$ in total.

Under a pay-as-you-go tariff $\boldsymbol{\theta^p}$, consumers pay p for each unit, and the reference price remains at p until the end of consumption. Therefore, the acquisition utility is $v(1-p)$ in each period. Hence, we can represent the total utility under a pay-as-you-go tariff, $V(\boldsymbol{\theta^p}(\mathbf{q_x}))$, as a function of $\boldsymbol{\theta^p}(\mathbf{q_x})$ as follows:

$$V(\boldsymbol{\theta^p}(\mathbf{q_x})) = x \cdot v(1 - p), \tag{12}$$

When consumers choose a tariff between the two, they evaluate the expected utility of both

and choose the one with higher expected utility. The expected utility under each tariff is obtained as follows:

$$V(\boldsymbol{\theta_n^f}) = E_x[V(\boldsymbol{\theta_n^f}, \mathbf{q_x})] = \sum_{i=1}^{\infty} V(\boldsymbol{\theta_n^f}, \mathbf{q_x}) \cdot Pr(x = i), \tag{13}$$

$$V(\boldsymbol{\theta^p}) = E_x[V(\boldsymbol{\theta^p}(\mathbf{q_x}))] = \sum_{i=1}^{\infty} V(\boldsymbol{\theta^p}(\mathbf{q_i})) \cdot Pr(x = i). \tag{14}$$

Then, we can obtain the following result.

Proposition 11. *A non-emotional consumer is indifferent between choosing a flat-rate tariff $\boldsymbol{\theta_n^f}$ and a pay-as-you-go tariff $\boldsymbol{\theta^p}$ when $n = E[x]$. In other words, $V(\boldsymbol{\theta_n^f}) = V(\boldsymbol{\theta^p})$ when $n = E[x]$.*

Table 4 shows examples of total expected utility for the non-emotional and the emotional consumer. For the non-emotional consumer, the expected utility under a pay-as-you-go tariff is higher than the expected utility under a flat-rate tariff if and only if the price is fair given the expected usage.

However, when consumers are emotional, the flat-rate tariffs become more attractive than the pay-as-you-go tariff, even if the prices are not fair given the expected usage ($n > 10$). For example, Case 1 shows that $V(\boldsymbol{\theta_{19}^f}) \le V(\boldsymbol{\theta^p}) \le V(\boldsymbol{\theta_{18}^f})$, which means that consumers would prefer a flat-rate tariff $\boldsymbol{\theta_{18}^f}$ to a pay-as-you-go tariff $\boldsymbol{\theta^p}$. Hence, consumers are willing to pay more to choose a flat-rate tariff and this can be considered a flat-rate bias as well. Also, the preference for a flat-rate tariff becomes stronger as the reference prices adapt more rapidly. Case 2 has a higher α, and therefore the reference prices adapts faster, and we can see that consumers would even prefer $\boldsymbol{\theta_{30}^f}$ to a pay-as-you-go tariff.

Consumers generally prefer a flat-rate tariff under demand uncertainty because of reference price adaptation. Under a flat-rate tariff, consumers have the risk of paying more when the realized demand is less than the units they have paid for, but as they start consuming the item, the feeling of using it for "free" offsets the disutility of having paid more. In addition, in case the realized demand is less than n, consumers incur disutility of $v(-(n - x)\hat{p}_x)$ in the last period x of consumption, but at that time the perceived price of the item $\hat{p}_x$ is lower than the actual price due to adaptation, mitigating the pain of feeling having paid more.

flat-rate (n)	Non-emotional	$\alpha = 0.3$	$\alpha = 0.5$
1	9.50	9.57	9.57
2	9.00	9.20	9.27
3	8.50	8.87	9.05
4	8.00	8.58	8.86
5	7.50	8.32	8.69
6	7.00	8.08	8.54
7	6.50	7.85	8.40
8	6.00	7.64	8.26
9	5.50	7.43	8.13
10	**5.00***	7.23	8.00
11	4.50	7.04	7.87
12	4.00	6.85	7.75
13	3.50	6.67	7.63
14	3.00	6.49	7.52
15	2.50	6.31	7.40
16	2.00	6.14	7.29
17	1.50	5.97	7.18
18	1.00	**5.80***	7.06
19	0.50	5.63	6.96
20	0.00	5.47	6.85
21	-0.50	5.31	6.74
22	-1.00	5.15	6.64
23	-1.50	4.99	6.53
24	-2.00	4.84	6.43
25	-2.50	4.68	6.33
26	-3.00	4.53	6.23
27	-3.50	4.38	6.13
28	-4.00	4.23	6.03
29	-4.50	4.08	5.93
30	-5.00	3.93	**5.83***
pay-as-you-go	5.00	5.74	5.74

Table 4: Expected total utility under demand uncertainty and RA for the non-emotional [$\lambda = \gamma = 1 - \alpha = 1$] and the non-emotional consumer [$\gamma = 0.8$ and $\lambda = 2$]. We fix $E[x] = 10$.

6.4 Tariff Switch

Lambrecht and Skiera (2006) observed that many consumers who were under a flat-rate tariff for Internet access could have saved money if they had switched to a pay-per-use tariff. However, their data showed that many of those people stuck to the flat-rate tariff persistently without switching to a pay-per-use tariff. In our model, we can observe that as consumers stay under a flat-rate tariff over multiple contract renewals, their perceived utility of a pay-as-you-go tariff decreases over time. In other words, as consumers get more used to a flat-rate tariff, they get less attracted to a pay-as-you-go tariff.

The reference price decays over time under a flat-rate tariff, whereas it remains at the unit price under a pay-as-you-go tariff. Therefore, when consumers renew their contract, their reference prices are lower when they were in a flat-rate tariff than when they were in a pay-as-you-go tariff in the previous contract. Let $V(\boldsymbol{\theta_1}, \hat{p}_1)$ and $V(\boldsymbol{\theta_T}, \hat{p}_1)$ be the total utility under a pay-as-you-go and a flat-rate tariff when the first reference price is $\hat{p}_1$. If a consumer's previous contract was a pay-as-you-go tariff, then $\hat{p}_1 = p$, and if her previous contract was a flat-rate tariff, then $\hat{p}_1 < p$. Then, we can obtain the following result.

Proposition 12. *Assume $v'(-x) \geq v'(x)$ for all $x > 0$. Then, when $0 \leq \hat{p}_1 \leq p$, $V(\boldsymbol{\theta_1}, \hat{p}_1)$ is increasing in $\hat{p}_1$. In other words, a consumer obtains higher utility under a pay-as-you-go tariff when the consumer's initial reference price is higher.*

Proposition 12 states that the total utility under a pay-as-you-go tariff increases as the initial reference price $\hat{p}_1$ increases. Put differently, if a consumer has a lower initial reference price, her utility under a pay-as-you-go tariff would be lower than someone whose initial reference price is higher. Therefore, if a consumer was under a flat-rate tariff in the previous contract, her reference price would have decreased below p, so her utility under a pay-as-you-go tariff would be lower than someone who was under a pay-as-you-go tariff in the previous contract. Based on this result, we can also observe the following.

Corollary 1. *When a consumer persistently stays under a flat-rate tariff through multiple contract renewals, the utility of choosing a pay-as-you-go tariff at the time of each renewal decreases over time.*

Corollary 1 shows that as a consumer stays under a flat-rate tariff over multiple contract renewals, a pay-as-you-go tariff would look less and less attractive at the time of each renewal opportunity. This result seems to partly explain why consumers persistently stayed under a flat-rate tariff over multiple contract renewal opportunities even when they could have saved money by switching to a pay-as-you-go tariff (Lambrecht and Skiera, 2006).

However, one can argue that although the utility of a pay-as-you-go tariff decreases as the reference price goes down, consumers would prefer to switch to a pay-as-you-go tariff as long as the utility of a pay-as-you-go tariff is higher than that of a flat-rate tariff. But in fact, even when $0 \leq \hat{p}_1 < p$, we can still show that consumers have the flat-rate bias.

Proposition 13. *Assume* $\lim_{x \to -\infty} v'(x) = 0$ *and* $v'(-x) \geq v'(x)$ *for all* $x > 0$. *When* $0 \leq \hat{p}_1 < p$, *there exists* T^* *such that for all* $T \geq T^*$, *consumers prefer a flat-rate tariff to a pay-as-you-go tariff if the reference price is updated by either RA or AFL.*

Previously, we showed that when $\hat{p}_1 = p$ consumers always prefer a flat-rate tariff to a pay-as-you-go tariff no matter what. Proposition 13 shows that the flat-rate bias still exists even when the initial reference price is lower than p, as long as the total period T is large enough. Therefore, our model could explain consumers' persistent loyalty for a flat-rate tariff over contract renewals.

Flat-rate contracts then pose a form of trap. Once the consumer is adapted to it, it becomes difficult to change to a pay-as-you-go tariff. An emotional consumer that foresees this will choose a flat rate if she does not foresee changing tariffs. Actual consumers, however, rarely predict correctly the magnitude of adaptation processes. In general, people predict at time t that the reference price at time τ will be somewhere between the current reference price, $\hat{p}_t$, and the actual reference price, $\hat{p}_\tau$ (**?**). This failure produces choices that fail to maximize experienced utility (Baucells and Sarin, 2010).

In the case of a preference for flat-rate, consumers will predict that their switching cost will be low to change to any other contract in the future if the choose to. In a way, they will have a false sense of future freedom. In actuality, the reference prices will adapt to not paying, and consumers will find it hard to change to a much more favorable pay-as-you-go contract.

7 Conclusion

Many studies have identified the existence of the flat-rate bias (Train, 1991) and tried to explain the phenomenon from different perspectives, such as prospective accounting (Prelec and Loewenstein, 1998; Lambrecht and Skiera, 2006) or demand uncertainty (Lambrecht and Skiera, 2006; Narayanan et al., 2007; DellaVigna and Malmendier, 2006; Grubb, 2009; Miravete, 2002, 2003; Goettler and Clay, 2009). We have introduced a model of double comparisons and reference price adaptation that offers a parsimonious and integrative account of the flat-rate bias, both under certainty and under uncertainty. The model predicts several other anomalies, all important and well documented, such as strong preference for advance payment, sunk-cost effect, payment depreciation, and tariff switching behaviors.

Our model can be considered a dynamic extension of the acquisition and transaction utility model of Thaler (1985). We adopt the concept of acquisition and transaction utility with minor modification. In the multi-period case, the reference price is intrinsically calculated, and the decision maker cannot help but use what his internal "adaptive" calculator proposes as a reference price. Hence, these consumers will experience hedonic feelings of "cheap or expensive" that are associated with payment methods.

There are several directions of future research. First, we could explore the contractual implications of this model from a firm's perspective. A hint of the implications has been given when discussing demand curves in the single period case. A more explicit investigation of a firm's optimal dynamic pricing policies over multiple periods is a direct application, which we leave for future research.

The model can be extended to new domains, such as supplier selection. Compare the option of choosing multiple suppliers (buy the plane ticket, book the hotel, rent a car) vs. one integrated supplier (a travel package provider). According to double accounting, people will exhibit a preference for one integrated supplier. With multiple suppliers, some prices will be above the reference price and others below. With an integrated supplier, there is a single comparison. Due to loss aversion, the integrated supplier will, on average, provide a higher hedonic value, even though the final cost may be more expensive.

Further experimental validation of the exact mechanism of double comparisons and reference

price adaptation would strengthen the theoretical value of this model. Moreover, some of the new effects predicted by the model should be better substantiated.

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A   Proofs

Proof of Proposition 1 If $v(x) = cx, c > 0$, then $v(u-\hat{p})+v(\hat{p}-p) = c\cdot(u-\hat{p})+c\cdot(\hat{p}-p) = c\cdot(u-p)$, which does not depend on the reference price $\hat{p}$. Conversely, set $x = u - \hat{p}$ and $y = \hat{p} - p$, and observe that f satisfies Pexider's equation:

$$f(x + y) = v_1(x) + v_2(y). \tag{15}$$

Because f is continuous at one point, necessarily, $f(x) = cx+a+b$, $v_1(x) = cx+a$ and $v_2(x) = cx+b$, where c, a, and b are arbitrary constants (Aczél, 1966). Because $v_1(0) = v_2(0) = 0$, $a = b = 0$, and $f(x) = v_1(x) = v_2(x) = v(x) = cx$. Because f is strictly increasing, $c > 0$.

3 implies 2. If $v(x) = cx$, then w solves $c(u - \hat{p}) + c(\hat{p} - w) = 0$, or $w = u$.

2 implies 1. If $w = u$, then □

Proof of Proposition 2 First, note that if the value function satisfies $v'(-x) > v'(x)$ for all $x > 0$, then it also satisfies that $-v(-x) > v(x)$ for all $x > 0$.

Define $V(\hat{p}, p) = v(u - \hat{p}) + v(\hat{p} - p)$ for a given u. By definition, the willingness to pay, w, satisfies $V(\hat{p}, w) = 0$. V is strictly decreasing with p. Moreover $V(\hat{p}, \hat{p}) = v(u - \hat{p}) + v(0) > 0$, and, because of loss aversion, $V(\hat{p}, u) = v(u - \hat{p}) + v(\hat{p} - u) < 0$. Hence, there is a unique $\hat{p} < w < u$ solving $V(\hat{p}, w) = 0$. Using the convexity of v for losses and loss aversion, respectively,

$$v'(\hat{p} - w) \geq v'(\hat{p} - u) > v'(u - \hat{p}).$$

By the implicit function theorem,

$$\frac{\partial w}{\partial \hat{p}} = -\frac{\partial V/\partial \hat{p}}{\partial V\, \partial w} = 1 - \frac{v'(u - \hat{p})}{v'(\hat{p} - w)} > 0. \quad \square$$

Proof of Proposition 3 The non-emotional consumer buys iff $u > p$. For the emotional consumer, we distinguish all the possible cases (the proposition omits some for simplicity):

If $u > p$ and $u \geq \hat{p} \geq p$, then $v(u-\hat{p}) \geq 0$ and $v(\hat{p}-p) \geq 0$, and one of the inequality is strict. Hence $w > p$.

If $\hat{p} < p < u$, then $v(u - \hat{p}) > 0$ and $v(\hat{p} - p) < 0$. If $p < u < \hat{p}$, then $v(u - \hat{p}) < 0$ and $v(\hat{p} - p) > 0$. In both cases, the sign of the sum is undecided.

If $u = p \neq \hat{p}$, then $v(u - \hat{p}) + v(\hat{p} - p) < 0$, because $|u - \hat{p}| = |\hat{p} - p|$ and loss aversion.

If $u = p = \hat{p}$, then $v(u - \hat{p}) = 0$ and $v(\hat{p} - p) = 0$, so the total utility is zero.

Let $u < p$. If $u \leq \hat{p} \leq p$, then $v(u - \hat{p}) \leq 0$ and $v(\hat{p} - p) \leq 0$, one of the two inequalities is strict. If $\hat{p} < u < p$, then $v(u - \hat{p}) > 0$ and $v(\hat{p} - p) < 0$. Because $u - \hat{p} < |\hat{p} - p|$ and loss aversion, $v(u - \hat{p}) < |v(\hat{p} - p)|$. If $u < p < \hat{p}$, then $v(u - \hat{p}) < 0$ and $v(\hat{p} - p) > 0$. Because $|u - \hat{p}| > \hat{p} - p$ and loss aversion, $|v(u - \hat{p})| > v(\hat{p} - p)$. In all three cases, the total utility is strictly negative. $\qquad\square$

Proof of Proposition 4 Define $V(u, p) = v(u - \hat{p}) + v(\hat{p} - p)$ for a given $\hat{p} > 0$. Let $\bar{u}(p)$ be a function such that $V(\bar{u}(p), p) = 0$ for any p. $\bar{u}(p)$ is well defined because V is strictly increasing in u, $V(0, p) < 0$, and $V(\infty, p) > 0$. Then, given a price p, consumers with $u \geq \bar{u}(p)$ would obtain nonnegative utility from consumption. Therefore, the fraction of consumers who obtain nonnegative utility when the price is p can be calculated as $d(p) = 1 - \bar{u}(p)$, since u is uniformly distributed in $[0, 1]$.

If $p < \hat{p}$, then it has to be the case that $\bar{u}(p) < \hat{p}$, and therefore $V(\bar{u}(p), p) = v(\bar{u}(p) - \hat{p}) + v(\hat{p} - p) = -\lambda(\hat{p} - \bar{u}(p))^\gamma + (\hat{p} - p)^\gamma = 0$. Hence, $\hat{p} - p = \lambda^{1/\gamma}(\hat{p} - \bar{u}(p))$ and $\bar{u}(p) = \hat{p} - (\hat{p} - p)/\lambda^{1/\gamma}$.

If $p \geq \hat{p}$, then it has to be the case that $\bar{u}(p) \geq \hat{p}$, and therefore $V(\bar{u}(p), p) = v(\bar{u}(p) - \hat{p}) + v(\hat{p} - p) = (\bar{u}(p) - \hat{p})^\gamma - \lambda(p - \hat{p})^\gamma = 0$. Hence, $\bar{u}(p) - \hat{p} = \lambda^{1/\gamma}(p - \hat{p})$ and $\bar{u}(p) = \hat{p} + (p - \hat{p})\lambda^{1/\gamma}$. $\qquad\square$

Proof of Proposition 5 If $v(x) = cx, c > 0$ and $\hat{p}_t = \hat{p}_1$, then

$$V = \sum_{t=1}^{T} \left[\delta^t u(q_t) - q_t \cdot \hat{p}_1 + \theta_t \cdot \hat{p}_1 - \delta'^t \cdot y_t \right] = \sum_{t=1}^{T} \left[\delta^t u(q_t) - \delta'^t \cdot y_t \right] + \hat{p}_1 \left(\sum_{t=1}^{T} \theta_t - \sum_{t=1}^{T} q_t \right).$$

Because, $\sum_{t=1}^{T} q_t = \sum_{t=1}^{T} \theta_t$, the result follows. For the converse, fix $\delta = \delta' = 1$. Let $T = 1$. By Proposition 1 we conclude that $f(x) = v(x) = cx, c > 0$. Let $T = 2$, $\theta_1 = 0$ and $\theta_2 = q_1 + q_2$. Then

$$
\begin{aligned}
V &= [u(q_1) - \hat{p}_1 q_1 - y_1] + [u(q_2) - \hat{p}_2 q_2 + \hat{p}_2(q_1 + q_2) - y_2] \\
&= u(q_1) - y_1 + u(q_2) - y_2 + (\hat{p}_2 - \hat{p}_1)q_1.
\end{aligned}
$$

If V is to be independent of the reference price, then $\hat{p}_2 = \hat{p}_1$. Assume $\hat{p}_t = \hat{p}_1, t = 1, ..., \tau - 1$. Let $T = \tau$, $\theta_t = 0, t = 1, ..., T - 1$, and $\theta_\tau = \sum_{t=1}^{T} q_t$. Then,

$$
\begin{aligned}
V &= \sum_{t=1}^{\tau-1} [u(q_t) - q_t \hat{p}_1 - y_t] + [u(q_\tau) - \hat{p}_\tau q_\tau + \hat{p}_\tau \sum_{t=1}^{\tau} q_t - y_\tau] \\
&= \sum_{t=1}^{\tau} [u(q_t) - y_t] + (\hat{p}_\tau - \hat{p}_1) \sum_{t=1}^{\tau-1} q_t.
\end{aligned}
$$

If V is to be independent of the reference prices, then $\hat{p}_\tau = \hat{p}_1$. By induction, $\hat{\mathbf{p}} = \mathbf{1} \cdot \hat{p}_1$. □

Proof of Proposition 6

i) Case 1: When the reference prices are updated by AFL, under a flat-rate tariff with prepayment $\boldsymbol{\theta_T}$, the reference prices are obtained as follows: $\hat{p}_1 = \hat{p}_2 = p$ and $\hat{p}_t = (1-\alpha)p$ for $3 \le t \le T$. The transaction utility occurs only in the first period, but $\hat{p}_1 = p$, so the transaction utility is zero. Therefore, $V(\boldsymbol{\theta_T}) = 2 \cdot v(1-p) + (T-2) \cdot v(1-(1-\alpha)p)$. Also, under a flat-rate tariff with postpayment $\boldsymbol{\theta'_T}$, the reference prices are obtained as follows: $\hat{p}_1 = p$ and $\hat{p}_t = (1-\alpha)p$ for $2 \le t \le T$. In this case, the transaction utility only occurs in period T, which is $v(T \cdot (1-\alpha)p - T \cdot p)$. Hence, $V(\boldsymbol{\theta'_T}) = v(1-p) + (T-1) \cdot v(1-(1-\alpha)p) + v(T((1-\alpha)p - p))$. Then,

$$V(\boldsymbol{\theta_T}) - V(\boldsymbol{\theta'_T}) = v(1-p) - v(1-p+\alpha p) - v(-\alpha p T). \tag{16}$$

Since we assumed $|v(-x)| \ge v(x)$ for all x, we can observe the following: $|v(-\alpha p T)| \ge v(\alpha p T) > v(\alpha p) > v(1-p) - v(1-p+\alpha p)$, where the last inequality holds because $v''(x) < 0$ when $x > 0$, and also because $p < 1$. Therefore, $V(\boldsymbol{\theta_T}) - V(\boldsymbol{\theta'_T}) = v(1-p) - v(1-p+\alpha p) - v(-\alpha p T) > 0$.

ii) Case 2: When the reference prices are updated by RA, under a flat-rate tariff with prepayment $\boldsymbol{\theta_T}$, the reference prices are obtained as follows: $\hat{p}_1 = \hat{p}_2 = p$ and $\hat{p}_t = (1-\alpha)^{t-2}p$ for $3 \le t \le T$. The transaction utility occurs only in the first period, but $\hat{p}_1 = p$, so the transaction utility is zero. Therefore, $V(\boldsymbol{\theta_T}) = 2 \cdot v(1-p) + \sum_{i=1}^{T-2} v(1-(1-\alpha)^i p)$. Also, under a flat-rate tariff with postpayment $\boldsymbol{\theta'_T}$, the reference prices are obtained as follows: $\hat{p}_1 = p$ and $\hat{p}_t = (1-\alpha)^{t-1}p$ for $2 \le t \le T$. In this case, the transaction utility only occurs in period T, which is $v(T \cdot (1-\alpha)^{T-1}p - T \cdot p)$. Hence, $V(\boldsymbol{\theta'_T}) = v(1-p) + \sum_{i=1}^{T-1} v(1-(1-\alpha)^i p) + v(T((1-\alpha)^{T-1}p - p))$. Then,

$$V(\boldsymbol{\theta_T}) - V(\boldsymbol{\theta'_T}) = v(1-p) - v(1-(1-\alpha)^{T-1}p) - v(T((1-\alpha)^{T-1}p - p)). \tag{17}$$

Since we assumed $|v(-x)| \ge v(x)$ for all x, we can observe the following: $|v(T((1-\alpha)^{T-1}p - p))| = |v(-T(p-(1-\alpha)^{T-1}p))| \ge v(T(p-(1-\alpha)^{T-1}p)) > v(p-(1-\alpha)^{T-1}p) > v(1-p) - v(1-(1-\alpha)^{T-1}p)$, where the last inequality holds because $v''(x) < 0$ when $x > 0$, and also because $p < 1$. Therefore, $V(\boldsymbol{\theta_T}) - V(\boldsymbol{\theta'_T}) = v(1-p) - v(1-(1-\alpha)^{T-1}p) - v(T((1-\alpha)^{T-1}p - p)) > 0$.

Proof of Proposition 7 When the total period is T, $\sum_{t=1}^{T} \theta_t = T$ and $\sum_{t=1}^{T} q_t < T$, the per-period utility is defined as follows:

$$V_t = v(u(q_t) - \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - y_t). \tag{18}$$

Hence, if a consumer forgoes consumption in period t (i.e. $q_t = 0$), the acquisition utility becomes $v(-\hat{p}_t)$. If the reference price is updated by RA, then $\hat{p}_{t+1} = (1-\alpha)\hat{p}_t + \alpha \bar{p}_t$, so we can observe

that $\hat{p}_{t+1} < \hat{p}_t$ for all $t \geq 2$. Therefore, the disutility of forgoing consumption decreases over time. However, if the reference price is updated by AFL, then $\hat{p}_{t+1} = (1 - \alpha)\hat{p}_1 + \alpha\bar{p}_t$. Hence, $\hat{p}_1 = \hat{p}_2 = p$ and $\hat{p}_t = (1 - \alpha)p$ for $t \geq 3$. Hence, the disutility decreases in the third period and persists over time.

Proof of Proposition 8 As argued, $V(\boldsymbol{\theta_1^b}) = n \cdot v(1 - p)$ and $V(\boldsymbol{\theta_T^b}) = \sum_{t=1}^{T} v(1 - \hat{p}_t)$. If $\alpha_{t,t} > 0$ and $t \geq 2$, then $\hat{p}_t < p$, $v(1 - \hat{p}_t) > v(1 - p)$, and the result follows. $\square$

Proof of Proposition 9 Under AFL, $\hat{p}_{t+1} = (1 - \alpha)\hat{p}_1 + \alpha\bar{p}_t$. Recall that $\hat{p}_1 = p$, and $\bar{p}_t = p$ if $\theta_t = 1$ and $\bar{p}_t = 0$ otherwise. The reference price is p in the first period and on periods $t = ni + 1, i = 1, ..., T/n$ following payment. On these $(1 + T/n)$ periods, the acquisition utility is $v(1 - p)$, and there is only the transaction utility in the first period, which is zero. On the remaining $T - 1 - T/n$ periods $\hat{p}_t = (1 - \alpha)p$. The acquisition utility is $v(1 - (1 - \alpha)p)$. Transaction utility, which occurs in $T/n - 1$ such periods, is $v((1 - \alpha)np - np) = v(-\alpha np)$. Hence,

$$
\begin{aligned}
V(\boldsymbol{\theta_n^b}) &= \sum_{t=1}^{T} v(1 - \hat{p}_t) + \sum_{i=1}^{T/n} v(\hat{p}_{(i-1)n+1} - np) \\
&= \left(1 + \frac{T}{n}\right) v(1 - p) + \left(T - 1 - \frac{T}{n}\right) v(1 - (1 - \alpha)p) + \left(\frac{T}{n} - 1\right) v(-\alpha np) \\
&= Tv(1 - (1 - \alpha)p) - v(-\alpha np) - \left(1 + \frac{T}{n}\right) [v(1 - (1 - \alpha)p) - v(1 - p)] + \frac{Tv(-\alpha np)}{n}.
\end{aligned}
$$

The first term does not depend on n. If $\alpha > 0$, the second and third terms are strictly increasing in n. The last term increases with n because, by the convexity of v for losses,

$$
v(-\alpha np) = v\left(-\frac{n}{n+1}\alpha(n+1)p\right) \leq \frac{n}{n+1}v(-\alpha(n+1)p). \quad \square
$$

Proof of Proposition 10 Considering the third term in (9), we have that

$$
\sum_{t=1}^{T} \int_0^{\hat{p}_t} f(u)v(u - \hat{p}_t)du \geq \sum_{t=1}^{T} v(-\hat{p}_t)P(u_t \leq \hat{p}_t) \geq -M.
$$

The term M is independent of T. If reference prices are minimally adaptive, the first two terms are strictly positive and non-decreasing with t. If T sufficiently large, their sum will exceed M.

If $P(u < (1 - \alpha)p) = 0$, then the third term is zero for $t \geq 3$, and hence (10) bounded. If prices are totally adaptive, then $\hat{p}_t$ will eventually be below ϵ. Because $P(u < \epsilon) = 0$, (10) is bounded. If RA holds, then $\hat{p}_t = (1 - \alpha)^{t-2}, t \geq 2$. If v is power, then

$$
\sum_{t=1}^{T} v(-\hat{p}_t)P(u_t \leq \hat{p}_t) \geq \sum_{t=1}^{T} v(-\hat{p}_t) = -\sum_{t=1}^{T} \lambda(1 - \alpha)^{\gamma(t-2)} \geq -\frac{\lambda}{(1 - \alpha)\ln\frac{1}{1-\alpha}}. \quad \square
$$

Proof of Proposition 11 Assume that the value function is $v(x) = x$ for all x, and the reference price $\hat{p}_t$ is equal to the actual price p for all t. Then, when the realized demand is x, a consumer's utility under a flat-rate tariff is obtained as follows:

$$V(\boldsymbol{\theta}_n^f, \mathbf{q_x}) = \begin{cases} \sum_{i=1}^x v(1 - \hat{p}_i) + v(-(n-x)\hat{p}_x) = x(1-p) - (n-x)p = x - np, & \text{if } x \leq n, \\ \sum_{i=1}^n v(1 - \hat{p}_i) + (x-n)v(1) = n(1-p) + (x-n) \cdot 1 = x - np, & \text{if } x > n. \end{cases} \quad (19)$$

Hence, the total utility is always $V(\boldsymbol{\theta}_n^f, \mathbf{q_x}) = x - np$, and the expected total utility is $V(\boldsymbol{\theta}_n^f) = E_x[V(\boldsymbol{\theta}_n^f, \mathbf{q_x})] = E_x[x - np] = E[x] - np$. Also, the total utility under a pay-as-you-go tariff is $V(\boldsymbol{\theta}^p(\mathbf{q_x})) = x \cdot v(1-p) = x(1-p)$, and therefore the expected total utility is $V(\boldsymbol{\theta}^p) = E_x[V(\boldsymbol{\theta}^p(\mathbf{q_x}))] = E_x[x(1-p)] = E[x](1-p)$. Therefore, we can observe that when $n = E[x]$, $V(\boldsymbol{\theta}_n^f) = V(\boldsymbol{\theta}^p)$.

Proof of Proposition 12 The total utility under a pay-as-you-go tariff when $\hat{p}_1 < p$ is obtained as follows:

$$V(\boldsymbol{\theta_1}, \hat{p}_1) = \sum_{t=1}^T \{v(u - \hat{p}_t) + v(\hat{p}_t - p)\}. \quad (20)$$

The derivative of this utility with respect to $\hat{p}_1$ is

$$\frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} = \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} + \sum_{t=2}^T \{-v'(u - \hat{p}_t) + v'(\hat{p}_t - p)\} \cdot \frac{\partial \hat{p}_t}{\partial \hat{p}_1}. \quad (21)$$

Since we assumed $v'(-x) \geq v'(x)$ for all $x > 0$, we can observe that $-v'(u - \hat{p}_t) + v'(\hat{p}_t - p) > 0$ for all $1 \leq t \leq T$, because $u - \hat{p}_t > |\hat{p}_t - p|$, and $v(x)$ is strictly convex when $x < 0$ and strictly concave when $x > 0$. Also, the reference price is defined as a weighted sum of past stimuli including $\hat{p}_1$, where all weights are nonnegative, it must be the case that $\frac{\partial \hat{p}_t}{\partial \hat{p}_1} \geq 0$ for all t. Hence,

$$\begin{aligned} \frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} &= \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} + \sum_{t=2}^T \{-v'(u - \hat{p}_t) + v'(\hat{p}_t - p)\} \cdot \frac{\partial \hat{p}_t}{\partial \hat{p}_1} \\ &\geq \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} > 0. \end{aligned} \quad (22)$$

Therefore, $\frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} > 0$.

Proof of Corollary 1 The reference price was defined as a weighted sum of the first reference and past stimuli as follows: $\hat{p}_{t+1} = \alpha_{t,0}\hat{p}_1 + \sum_{\tau=1}^t \alpha_{t,\tau}\bar{p}_\tau$, where $\alpha_{t,t} > 0$. Under a flat-rate tariff, since $\bar{p}_\tau = 0$ for all $2 \leq \tau \leq T$, the reference price at the end of the consumption stream, $\hat{p}_T$, is lower than the first reference price $\hat{p}_1$. When consumers renew their contract, the new first reference price $\hat{p}_1$ is equivalent to the last reference price of their previous contract. Therefore, when consumers

stay under a flat-rate tariff through multiple contract renewals, their initial reference price at the beginning of each renewal decreases over time. Hence, by Proposition 12, the utility of choosing a pay-as-you-go tariff at the time of each renewal decreases over time.

Proof of Proposition 13

i) Case 1: Assume the reference price is updated by AFL. Then, the total utilities of a pay-as-you-go and a flat-rate tariff are obtained as follows:

$$V(\boldsymbol{\theta_1}, \hat{p}_1) = \sum_{t=1}^{T} \{v(u - \hat{p}_t) + v(\hat{p}_t - p)\} \tag{23}$$

$$= \{v(u - \hat{p}_1) + v(\hat{p}_1 - p)\} + (T - 1)\{v(u - ((1 - \alpha)\hat{p}_1 + \alpha p)) + v(((1 - \alpha)\hat{p}_1 + \alpha p) - p)\}, \tag{24}$$

$$V(\boldsymbol{\theta_T}, \hat{p}_1) = \sum_{t=1}^{T} v(u - \hat{p}_t) + v(T\hat{p}_1 - Tp) \tag{25}$$

$$= v(u - \hat{p}_1) + v(u - ((1 - \alpha)\hat{p}_1 + \alpha p)) + (T - 2)v(u - (1 - \alpha)\hat{p}_1) + v(T(\hat{p}_1 - p)). \tag{26}$$

The difference between the two utilities is

$$V(\boldsymbol{\theta_T}, \hat{p}_1) - V(\boldsymbol{\theta_1}, \hat{p}_1) = (T - 2)\{v(u - (1 - \alpha)\hat{p}_1) - v(u - ((1 - \alpha)\hat{p}_1 + \alpha p))\} \tag{27}$$

$$+ v(T(\hat{p}_1 - p)) - v(\hat{p}_1 - p) - v(((1 - \alpha)\hat{p}_1 + \alpha p) - p). \tag{28}$$

We can substitute the following: $\epsilon_1 = \{v(u - (1 - \alpha)\hat{p}_1) - v(u - ((1 - \alpha)\hat{p}_1 + \alpha p))\} > 0$ and $\epsilon_2 = -\{v(\hat{p}_1 - p) + v(((1 - \alpha)\hat{p}_1 + \alpha p) - p)\} > 0$. Note that ϵ_1 and ϵ_2 are both positive. Then, the difference can be simply represented as

$$V(\boldsymbol{\theta_T}, \hat{p}_1) - V(\boldsymbol{\theta_1}, \hat{p}_1) = (T - 2)\epsilon_1 + v(T(\hat{p}_1 - p)) + \epsilon_2. \tag{29}$$

Note that the first term $(T - 2)\epsilon_1$ is positive and linearly increasing in T, and the second term $v(T(\hat{p}_1 - p))$ is negative and decreasing in T. Since we assumed $\lim_{x \to -\infty} v'(x) = 0$, we can observe that $\lim_{T \to \infty} v'(T(\hat{p}_1 - p)) = 0$, and hence there exists T^* such that for all $T \geq T^*$, $(T - 2)\epsilon_1 \geq |v(T(\hat{p}_1 - p))|$. Therefore, there exists T^* such that for all $T \geq T^*$, $V(\boldsymbol{\theta_T}, \hat{p}_1) > V(\boldsymbol{\theta_1}, \hat{p}_1)$.

ii) Case 2: Assume the reference price is updated by RA. In this proof, for ease of exposition, we define $\boldsymbol{\theta_{n,T}}$ as $\boldsymbol{\theta_n}$ where the total period is T. Then, $\boldsymbol{\theta_{T,T}}$ and $\boldsymbol{\theta_{1,T}}$ are a flat-rate tariff and a pay-as-you-go tariff, respectively, when the total period is T. Also, $V(\boldsymbol{\theta_{T,T}})$ and $V(\boldsymbol{\theta_{1,T}})$ are the total utility under each tariff, respectively. Now, define $D(T) = V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{1,T}})$, which represents the difference of utility between a flat-rate tariff and a pay-as-you-go tariff. For convenience, let $D(0) = 0$. Also, define $\Delta_T = D(T) - D(T-1)$. Let $\hat{p}_t$ and $\hat{p}'_t$ be the reference prices with a flat-rate

tariff and a pay-as-you-go tariff, respectively. Then,

$$\Delta_T = D(T) - D(T-1) \tag{30}$$

$$= \{V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{T-1,T-1}})\} - \{V(\boldsymbol{\theta_{1,T}}) - V(\boldsymbol{\theta_{1,T-1}})\} \tag{31}$$

$$= \{v(u - \hat{p}_T) + v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\} - \{v(u - \hat{p}'_T) + v(\hat{p}'_T - p)\} \tag{32}$$

$$> \{v(u - \hat{p}_T) + v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\} - v(u - p), \tag{33}$$

since $v(u - \hat{p}'_T) + v(\hat{p}'_T - p)$ is increasing in $\hat{p}'_t$ by Proposition 12 and $\hat{p}'_t < p$. Rearranging the inequality above, we obtain

$$\Delta_T > \{v(u - \hat{p}_T) - v(u - p)\} + \{v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\}, \tag{34}$$

and we call the RHS the lower bound of Δ_T. Note that $v(u - \hat{p}_T) - v(u - p) > 0$ and this is (weakly) increasing in T, because $\hat{p}_T < p$ and $\hat{p}_T$ is (weakly) decreasing under a flat-rate tariff. Also, the second term is negative but it is increasing in T due to diminishing sensitivity. Therefore the lower bound of Δ_T is a sum of two increasing functions and hence it is also increasing. In addition, since we assume $\lim_{x \to -\infty} v'(x) = 0$, for any $\delta > 0$, there exists T_0 such that for all $T \geq T_0$, $|v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))| < \delta$. Therefore, there exists T_1 such that for all $T \geq T_1$, the lower bound of Δ_T is positive. Let ϵ be the lower bound of Δ_{T_1}. Then, for all $T \geq T_1$, $\Delta_{T_1} > \epsilon$.

For any $T > T_1$,

$$D(T) = \sum_{t=1}^{T} \Delta_t = \sum_{t=1}^{T_1-1} \Delta_t + \sum_{t=T_1}^{T} \Delta_t > D(T_1 - 1) + (T - T_1)\epsilon. \tag{35}$$

Since $D(T_1 - 1)$ is a finite number and $(T - T_1)\epsilon > 0$ is linearly increasing in T, there exists T_2 such that for all $T \geq T_2$, $D(T) > D(T_1 - 1) + (T - T_1)\epsilon > 0$. Since $D(T) = V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{1,T}}) > 0$, when $T \geq T_2$, $V(\boldsymbol{\theta_{T,T}}) > V(\boldsymbol{\theta_{1,T}})$.